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BUCKLING LOADS FOR TAPERED SLENDER
ELASTIC COLUMNS

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BUCKLING LOADS FOR TAPERED
SLENDER ELASTIC COLUMNS

by

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ABSTRACT

Elastic buckling loads are obtained for a family of tapered columns using the Vianello iteration method as refined by Newmark. The family of columns studied by Dinnik is generalized to include columns not symmetric about mid-length. An extensive table of buckling factors is compiled with the aid of a high speed digital computer. Some of Dinnik's results are found to be in error.

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LIST OF SYMBOLS

A		L
A_1	- Arbitrary constants of integration in Appendix B	$L^{1/2}$
A_2		-
a	- Length of prismatic portion of column	L
B		L
B_1	- Arbitrary constants of integration in Appendix B	$L^{1/2}$
B_2		-
b	- Distance from point of initiation of taper to left end of column	L
c	- Distance from left end of column to center of prismatic portion	L
D	- The integral $\int_0^l \frac{I_2}{I_x} y^2 dx$, Rayleigh method	L^3
d	- Distance from right end of prismatic portion to the right end of column	L
E	- Modulus of elasticity	FL^{-2}
e	- Distance from right end of column to point of termination of taper	L
g	- Distance from left end of column to left end of prismatic portion	L
h	- Dimensionless integration factor	-
h_1	- The smallest value of h	-
I	- Minimum moment of inertia of column cross-sectional area	L^4
I_1	- I at smallest column cross-section	L^4
I_2	- I at largest column cross-section	L^4
I_x	- I at distance x	L^4
j	- Distance from point of initiation of taper to point of maximum cross sectional area ($=b+g$)	L
K	- Dimensionless ratio ($= (I_1/I_2)^{1/n}$)	-

k	- Number of equal parts into which the column length is divided for numerical integration	-
ℓ	- Column length	L
m	- Buckling factor ($= P \ell^2 / EI_2$)	-
N	- The integral $\int_0^{\ell} (y')^2 dx$, Rayleigh method	L ³
n	- Exponent characterizing degree of taper	-
P	- Buckling load	F
p	- Interpolation factor	-
q	- Ordinate of curvature loading curve	L ⁻¹
r, r_1	- Ordinates of curvature loading curve	L ⁻¹
s, s_1	- Ordinates of curvature loading curve	L ⁻¹
T	- Work done by external load during column buckling	FL
t	- Integration factor ($= 1/x^*$)	L ⁻¹
u	- Dimensionless longitudinal coordinate ($= 1 - 2x/\ell$)	-
V	- Increase of strain energy accompanying buckling	FL
v	- Decrease in distance between ends of column during loading	L
x	- Longitudinal coordinate	L
x^*	- Longitudinal coordinate ($= x + b$)	L
y	- Transverse deflection	L
α	- Integration factor ($= \sqrt{\frac{PJ^2}{EI_2}}$)	-
β	- Ratio in analytical solution ($= \frac{\delta \ell}{2bv}$)	-
γ	- Ratio in analytical solution $= \frac{1}{2} h_1 \ell n(I_1/I_2)$	-
δ	- Integration factor ($= \sqrt{\frac{PJ^2}{EI_2}}$)	L
λ	- Length interval used in numerical integration	L
ξ	- Dimensionless longitudinal coordinate ($= \frac{x^*}{\ell}$)	-

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1. Introduction

For an initially-straight slender column which obeys Hooke's law, the buckling load is defined to be the concentric compressive load under which the straight configuration becomes neutrally stable. The buckling load is a function of geometry, material, and end conditions, and it takes the form:

$$P = \frac{m EI_c}{\ell^2} \quad (1.1)$$

where P is the buckling load, m is the buckling factor, E is the modulus of elasticity of the column material, I is the minimum centroidal moment of inertia of the column cross sectional area, and ℓ is the column length.

Columns with varying cross-sections are often used when the prismatic column is inadequate. The varying cross-section column is used in many applications in order to decrease weight or provide additional strength. Early studies of the effect of varying cross-sections on buckling loads were done by Dinnik (1) and by Bairstow and Stedman (2). Dinnik presented a table for columns which tapered symmetrically about a central prismatic section. He based his analytical calculations on the assumption of various power laws for the stiffness in the end portions of the columns.

Bairstow and Stedman presented a numerical method of solving for buckling loads in columns with varying cross-sections. This method was later improved by Ratzersdorfer (3). Vianello (4) presented a graphical method of solution for buckling loads, and Morley (5) introduced numerical integration into Vianello's

process. Newmark (6) introduced a number of refinements to this process and summarized the entire procedure. Approximate solutions based on the energy method were given by Bleich (7), and Boyd (8) conducted both theoretical and experimental studies on tapered columns.

The present study uses a numerical method of solution to find elastic buckling loads for columns of varying cross-section. Specifically, the columns are tapered, hinge-ended, slender, initially-straight, and concentrically loaded. A large family of columns is studied, and Dinnik's columns, tapered symmetrically about mid-length, are members of this family. Dinnik's data are checked and augmented by obtaining additional points and by providing four significant figure results. The major portion of the results, however, provides buckling loads for columns which are not symmetric about midlength. These latter results are believed to be new.

The work can best be described by dividing it into three phases: selection of a numerical method of solution; adaptation and use of the selected method in solving for the buckling loads; and discussion and verification of results.

2. Selection of a Numerical Method

The selection of a numerical method involved the investigation of the Bairstow-Stedman, the Vianello, the Rayleigh (9), and the Rayleigh-Ritz (10) methods. Each of these methods is designed to find the buckling factor for a varying cross-section column (11).

In the discussion of the four methods, it is assumed that I_x , the moment of inertia at a longitudinal distance x , is known at all points. It is further assumed that the boundary conditions are those of a hinged-end column:

$$y = 0 \quad \text{at} \quad x = 0 \quad (2.1)$$

$$y = 0 \quad \text{at} \quad x = \ell \quad (2.2)$$

Both the Bairstow-Stedman and the Vianello methods utilize the differential equation for the deflection curve of a slender column under a compressive load and with small deflections:

$$EI_x \frac{d^2 y}{dx^2} + Py = 0 \quad (2.3)$$

where y is the lateral deflection at the longitudinal distance x . It is convenient to modify this equation by a form of equation (1.1):

$$P = \frac{mEI_2}{\ell^2}$$

where I_2 is chosen to be the maximum moment of inertia in the column. Equation (2.3) then becomes:

$$\frac{d^2 y}{dx^2} + \frac{m}{\ell^2} \left(\frac{I_2}{I_x} \right) y = 0 \quad (2.4)$$

In the Bairstow-Stedman method, a value for m is assumed, and after the quantity mI_2/I_x is calculated, the following approximate equations are used in a numerical integration:

$$y_i = y_{i-1} + \lambda y'_{i-\frac{1}{2}} \quad (2.5)$$

$$y''_i = \frac{-m}{\lambda^2 k^2} \left(\frac{I_2}{I_x} \right) y_i \quad (2.6)$$

$$y'_{i+\frac{1}{2}} = y'_{i-\frac{1}{2}} + \lambda y''_i \quad (2.7)$$

The primes indicate differentiation with respect to x , and $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$. The column is divided into k increments of equal length λ , or $\ell = k\lambda$. The subscript i indicates the longitudinal position and is equal to kx/ℓ , i.e., when $i = 0$, $x = 0$ and when $i = k$, $x = \ell$. The initial entries y_0 and y''_0 are equal to zero in accordance with the first boundary condition and equation (2.6). In order to begin the integration an initial value for $y'_{1/2}$ at $x = \lambda/2$ is arbitrarily assumed. Then the equations (2.5), (2.6), and (2.7) are successively employed until the integration has proceeded along the entire length of the column. If, at the completion of integration, the deflection corresponds to the second boundary condition, i.e., $y_k = 0$, the correct value of m was assumed. If the end deflection does not meet the boundary condition, new values of m must be assumed until the second boundary condition is met.

In the Vianello method, the initial deflection curve is assumed. Then, using equations (2.6), (2.7), and (2.5), respectively, the curve yk^2/m is obtained by numerical integration. By dividing, point by point, the assumed deflection curve by this value and multiplying by the known value k^2 , a value of m for each point is obtained. If all values of m agree, the correct deflection curve was assumed. If they do not agree, the curve

y''^2/m is assumed to be the deflection curve, and the process is repeated until all values of m are the same. Since each assumed deflection curve is known to be more accurate than the preceding one (6), the Vianello process will eventually converge to a stable value of m .

The Rayleigh method makes direct use of the fact that a column is in a state of neutral equilibrium when it is subjected to the buckling load. This can be expressed by the condition $T = V$. T is the work done by external loads as the column is displaced from a straight to a bent configuration and V is the strain energy of bending induced in the column as it is displaced. Additionally, $T = Pv$, where v is the decrease in distance between the ends of the column as the column assumes the deflection curve y . For small displacements, v is approximated by:

$$v = \frac{1}{2} \int_0^l (y')^2 dx, \quad \text{and so}$$

$$T = \frac{1}{2} P \int_0^l (y')^2 dx$$

V may be expressed as $\int_0^l \frac{M^2 dx}{2EI_x}$ where M is the bending moment.

Since $M = Py$,

$$V = \frac{1}{2} P^2 \int_0^l \frac{y^2 dx}{EI_x}$$

Equating T and V and solving for P ,

$$P = \frac{\int_0^l (y')^2 dx}{\int_0^l \frac{y^2 dx}{EI_x}}$$

or,

$$m = \frac{\int_0^l (y')^2 dx}{\int_0^l \frac{I_2}{I_x} y^2 dx} = \frac{N}{D} \quad (2.8)$$

The solution for m consists of assuming a deflection curve $y(x)$ which satisfies the boundary conditions, equations (2.1) and (2.2), and integrating to find N and D . It is generally impractical to evaluate D analytically; a numerical integration procedure is more convenient. An accurate value of m can be obtained by applying this procedure to a number of deflection curves. Of these, the correct or most accurate deflection curve yields the minimum value of m .

The main advantages of this system are speed and simplicity of calculation. The choice of a family of deflection curves which can be varied to produce the correct deflection curve is difficult, however, and becomes even more so with the introduction of the non-symmetric column.

Ritz provided an extension of the Rayleigh method which eliminates the previous objection. He employed equation (2.8) in devising a method to adjust systematically the parameters in an assumed deflection curve to yield a minimum value of m . The method, however, employs a comparatively large number of separate numerical integrations which would become quite cumbersome in solutions for m in non-symmetric columns.

The investigation of the four methods was not intended to be exhaustive. Since each method will yield a solution, the selection of the method to be used depended only upon the relative adaptability of the methods to the present study. The Vianello method with its automatic convergence was judged to be most suitable to the present study and is particularly adaptable to non-symmetric columns. The procedure outlined by Newmark

and the refinements which he introduced for improved accuracy were used in the solutions. These refinements are presented in Appendix A.

3. Solution for the Buckling Factor

The family of columns studied is an extension of that considered by Dinnik. The form is shown in Fig. 1. The moment of inertia of cross-sectional area at distance x from the left-hand end is given by:

$$\frac{I_x}{I_2} = \left(\frac{x/l + b/l}{g/l + b/l} \right)^n \quad 0 \leq x/l \leq g/l$$

$$\frac{I_x}{I_2} = 1.0 \quad g/l \leq x/l \leq (g/l + a/l)$$

$$\frac{I_x}{I_2} = \left(\frac{1 + e/l - x/l}{d/l + e/l} \right)^n \quad (g/l + a/l) \leq x/l \leq 1.0$$

Dinnik restricted his study to columns symmetric about midlength, i.e., $b = e$, $g = d$, $c = l/2$. In the present study, non-symmetric columns are also included. The shape of an individual column is uniquely determined by specifying I_1/I_2 , n , a/l , and c/l . It is easily shown that the other parameters are given by:

$$g/l = c/l - a/2l$$

$$d/l = 1 - g/l - a/l$$

$$b/l = \left(\frac{K}{1-K} \right) g/l$$

$$e/l = \left(\frac{K}{1-K} \right) d/l$$

where $K = (I_1/I_2)^{1/n}$

It is of interest to note that the postulated variation of I is attainable through linear taper for $n = 1, 2, 3$, and 4 .

Thus: $n = 1$ corresponds to a column of rectangular cross-section

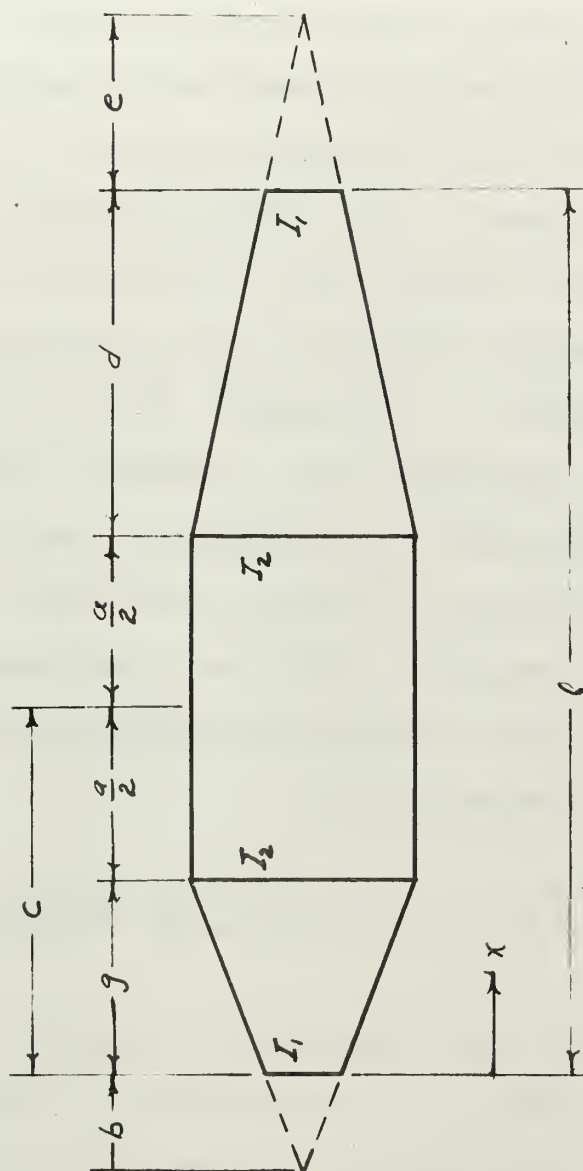


Figure 1 Column Under Study

with constant depth and linearly varying width; $n = 2$ is approximated by a built-up column consisting of four angles connected by diagonals; $n = 3$ corresponds to a rectangular cross-section column with constant width and linearly varying depth; and $n = 4$ is represented by a solid truncated cone or pyramid.

Since the Vianello method automatically converges, the assumed shape of the initial deflection curve is not critical. However, the choice deserves some attention because the number of iterations required depends upon the accuracy of the starting assumption. The boundary conditions for a hinge-ended column require zero deflection at the two ends. When I_1/I_2 is unity, the tapered column degenerates into a prismatic column with a sinusoidal deflection curve. From preliminary calculations and observations of the symmetric columns, it was noted that as I_1/I_2 becomes quite small, the deflection curve becomes approximately parabolic. With these factors in mind, the initial deflection curve is assumed to be:

$$y = (1 - u^2)(1 - I_1/I_2) + I_1/I_2 \sin\left(\frac{\pi x}{\ell}\right) \quad (3.1)$$

where $u = 1 - 2x/\ell$. Since the amount of variation in the deflection curve caused by the introduction of a non-symmetric column was unknown, the variation is ignored in equation (3.1).

The general solution used to find the critical buckling factor can be best explained by an example, Table I. A typical non-symmetric column is chosen with $n = 2$, $c/\ell = 0.25$, $a/\ell = 0.0$, and $I_1/I_2 = 0.5$. For this example, the length is divided into 8 segments, i.e., $\lambda = 0.125$ and $k = 8$.

TABLE I - EXAMPLE OF GENERAL SOLUTION

①	x/ρ	0.0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1.0
②	I_x/I_2	0.5	0.73	1.0	0.95	0.81	0.72	0.65	0.57	0.5
③	u	1.0	0.75	0.5	0.25	0.0	-0.25	-0.5	-0.75	1.0
④	$1-u^2$	0.0	0.44	0.75	0.94	1.0	0.94	0.75	0.44	0.0
⑤	$\sin \pi x$	0.0	0.38	0.71	0.98	1.0	0.98	0.71	0.38	0.0
⑥	y	0.0	0.41	0.73	0.96	1.0	0.96	0.73	0.41	0.0
⑦	$\frac{y''k^2\lambda^2}{m}$	0.0	-0.56	-0.73	-1.01	-1.24	-1.33	-1.12	-0.72	0.0
⑧	$\frac{\Delta y'k^2\lambda}{m}$	0.0	-0.53	-0.77	-1.00	-1.23	-1.31	-1.10	-0.69	0.0
⑨	$\frac{y'k^2\lambda}{m}$	2.91	2.38	1.61	0.61	-0.62	-1.93	-3.03	-3.72	
⑩	$\frac{yk^2}{m}(1)$	0.0	2.91	5.29	6.90	7.51	6.89	4.96	1.93	-1.79
⑪	correc- tion	0.0	0.22	0.45	0.67	0.90	1.12	1.34	1.57	1.79
⑫	$\frac{yk^2}{m}$	0.0	3.13	5.74	7.57	8.41	8.01	6.30	3.50	0.0
⑬	m	-	8.38	8.14	8.11	7.61	7.67	7.41	7.50	-

The computed values of I_x/I_2 are entered on Line 2. Lines 3, 4, and 5 are self-explanatory. Line 6 is the assumed value of the deflection curve of the column when subjected to the buckling load. Line 7 was obtained by employing equation (2.6).

$$y_i'' = \frac{-m}{\lambda^2 R^2} \left(\frac{I_2}{I_x} \right) y_i \quad (2.6)$$

Line 7 is the negative of the quotient of the corresponding values on Line 6 by those on Line 2. Line 8 is the result of altering Line 7 with a refinement by Newmark. This refinement is explained in Appendix A. In a symmetric column, the slope (y') is zero at midlength. For a non-symmetric column, this condition is not met, but it is nevertheless used to obtain an initial value for Line 9. Using this assumption, the initial value on Line 9 at $x = 0.45\ell$ is set equal to one half the negative of the entry at $x = 0.5\ell$ on Line 8. The entries to the left of $x = 0.45\ell$ are determined from right to left by successive subtraction of the intervening numbers of Line 8. Line 10 is determined by employing equation (2.5).

$$y_i = y_{i-1} + \lambda y'_{i-\frac{1}{2}} \quad (2.5)$$

The initial value is obtained from the boundary condition that $y = 0$ at $x = 0$. The following entries are the cumulative sum of the entries of Line 9. The incorrect choice of the position of zero slope and the incorrect assumption of a symmetric deflection curve lead to a non zero entry at $x = \ell$ on Line 10. The second boundary condition specifies that this value be zero.

Line 11 provides a linear correction to Line 10 enabling the boundary condition to be met. The entries of Line 11 are computed by multiplying the negative of the $x = \ell$ entry of Line 10 by the respective entries of Line 1. The entries of Line 12 are the sum of the corresponding values of Lines 10 and 11. Finally, the values of m on Line 13 are the quotient of the corresponding entries on Line 6 by those on Line 12 multiplied by k^2 (64).

Since it is desired to determine m to four significant figures (three decimal places), the criterion for convergence is that the maximum and minimum values of m on Line 13 be within 0.0005 of each other. If, as in Table I, this has not occurred, Line 12 becomes the new assumed deflection curve (Line 6) and the process is repeated to convergence. At convergence, the value of m is taken to be the average of the maximum and minimum entries of Line 13.

This general method of solution was programmed for the CDC 1604 high speed digital computer. The factors a/ℓ , c/ℓ , I_1/I_2 , and n were varied to cover the usable range of columns of this type, and the resulting buckling factors are listed in Table II. The factor n varies from 1 to 4 in integer steps. For each n , c/ℓ varies from 0.00 to 0.50 in increments of 0.05; it should be noted that values of c/ℓ from 0.5 to 1.0 are merely repetitions of those from 0.5 to 0.0. For each c/ℓ , I_1/I_2 varies from 0.05 to 1.00 in increments of 0.05 and a/ℓ varies from 0.0 to $2c/\ell$ in increments of 0.1. It should be noted that a column is not defined when $a/2\ell$ is greater than c/ℓ .

TABLE II
CRITICAL BUCKLING FACTOR

		G/L=0.00											
		N=1.0											
		A/L											
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
11/12													
.05	*	4.231	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.10		4.667	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.15		5.054	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.20		5.411	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.25		5.748	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.30		6.069	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.35		6.379	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.40		6.679	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.45		6.971	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.50		7.256	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.55		7.535	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.60		7.809	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.65		8.078	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.70		8.343	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.75		8.605	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.80		8.864	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.85		9.119	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.90		9.372	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
.95		9.622	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
1.00		9.870	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	

*.000 given for "impossible" columns ($a/2l > c/l$)

TABLE II (CONTINUED)
N=1.0 C/L=0.05

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11/12											
.05	4.418	4.627	.000	.000	.000	.000	.000	.000	.000	.000	.00
.10	4.849	5.051	.000	.000	.000	.000	.000	.000	.000	.000	.00
.15	5.229	5.425	.000	.000	.000	.000	.000	.000	.000	.000	.00
.20	5.580	5.767	.000	.000	.000	.000	.000	.000	.000	.000	.00
.25	5.909	6.088	.000	.000	.000	.000	.000	.000	.000	.000	.00
.30	6.222	6.393	.000	.000	.000	.000	.000	.000	.000	.000	.00
.35	6.523	6.684	.000	.000	.000	.000	.000	.000	.000	.000	.00
.40	6.814	6.965	.000	.000	.000	.000	.000	.000	.000	.000	.00
.45	7.096	7.237	.000	.000	.000	.000	.000	.000	.000	.000	.00
.50	7.372	7.501	.000	.000	.000	.000	.000	.000	.000	.000	.00
.55	7.640	7.758	.000	.000	.000	.000	.000	.000	.000	.000	.00
.60	7.904	8.010	.000	.000	.000	.000	.000	.000	.000	.000	.00
.65	8.162	8.256	.000	.000	.000	.000	.000	.000	.000	.000	.00
.70	8.416	8.498	.000	.000	.000	.000	.000	.000	.000	.000	.00
.75	8.667	8.735	.000	.000	.000	.000	.000	.000	.000	.000	.00
.80	8.913	8.969	.000	.000	.000	.000	.000	.000	.000	.000	.00
.85	9.157	9.199	.000	.000	.000	.000	.000	.000	.000	.000	.00
.90	9.397	9.425	.000	.000	.000	.000	.000	.000	.000	.000	.00
.95	9.635	9.649	.000	.000	.000	.000	.000	.000	.000	.000	.00
1.00	9.870	9.870	.000	.000	.000	.000	.000	.000	.000	.000	.00

TABLE II (CONTINUED)

11/12	N=1.0											
	C/L=0.10											
	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
.05	4.623	4.828	5.116	.000	.000	.000	.000	.000	.000	.000	.000	.000
.10	5.047	5.275	5.525	.000	.000	.000	.000	.000	.000	.000	.000	.000
.15	5.420	5.640	5.881	.000	.000	.000	.000	.000	.000	.000	.000	.000
.20	5.762	5.974	6.204	.000	.000	.000	.000	.000	.000	.000	.000	.000
.25	6.083	6.285	6.505	.000	.000	.000	.000	.000	.000	.000	.000	.000
.30	6.387	6.580	6.787	.000	.000	.000	.000	.000	.000	.000	.000	.000
.35	6.679	6.860	7.056	.000	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.960	7.130	7.313	.000	.000	.000	.000	.000	.000	.000	.000	.000
.45	7.232	7.390	7.560	.000	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.496	7.642	7.798	.000	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.753	7.887	8.029	.000	.000	.000	.000	.000	.000	.000	.000	.000
.60	8.005	8.125	8.253	.000	.000	.000	.000	.000	.000	.000	.000	.000
.65	8.252	8.359	8.471	.000	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.494	8.586	8.684	.000	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.732	8.810	8.892	.000	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.966	9.029	9.095	.000	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.196	9.244	9.295	.000	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.424	9.456	9.490	.000	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.648	9.664	9.681	.000	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)
N=1.0 C/L=0.15

	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	4.845	5.112	5.404	5.726	.000	.000	.000	.000	.000	.000	.000	.000
.10	5.260	5.520	5.801	6.109	.000	.000	.000	.000	.000	.000	.000	.000
.15	5.624	5.875	6.146	6.440	.000	.000	.000	.000	.000	.000	.000	.000
.20	5.957	6.199	6.457	6.736	.000	.000	.000	.000	.000	.000	.000	.000
.25	6.268	6.499	6.745	7.009	.000	.000	.000	.000	.000	.000	.000	.000
.30	6.562	6.782	7.014	7.263	.000	.000	.000	.000	.000	.000	.000	.000
.35	6.843	7.050	7.269	7.502	.000	.000	.000	.000	.000	.000	.000	.000
.40	7.112	7.307	7.511	7.728	.000	.000	.000	.000	.000	.000	.000	.000
.45	7.373	7.554	7.743	7.943	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.625	7.793	7.967	8.150	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.871	8.024	8.182	8.348	.000	.000	.000	.000	.000	.000	.000	.000
.60	8.111	8.249	8.390	8.538	.000	.000	.000	.000	.000	.000	.000	.000
.65	8.345	8.467	8.592	8.723	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.574	8.680	8.789	8.901	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.799	8.889	8.980	9.074	.000	.000	.000	.000	.000	.000	.000	.000
.80	9.020	9.093	9.166	9.241	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.237	9.292	9.348	9.405	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.451	9.488	9.526	9.564	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.662	9.681	9.699	9.718	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

I1/I2	N=1.0										
	C/L=0.20										
	A/L										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.05	5.081	5.358	5.720	6.082	6.476	.000	.000	.000	.000	.000	.000
.10	5.485	5.783	6.103	6.447	6.819	.000	.000	.000	.000	.000	.000
.15	5.838	6.127	6.433	6.760	7.110	.000	.000	.000	.000	.000	.000
.20	6.160	6.437	6.730	7.039	7.367	.000	.000	.000	.000	.000	.000
.25	6.459	6.725	7.002	7.294	7.601	.000	.000	.000	.000	.000	.000
.30	6.741	6.994	7.256	7.529	7.815	.000	.000	.000	.000	.000	.000
.35	7.010	7.249	7.495	7.750	8.015	.000	.000	.000	.000	.000	.000
.40	7.268	7.492	7.721	7.958	8.201	.000	.000	.000	.000	.000	.000
.45	7.516	7.725	7.937	8.154	8.377	.000	.000	.000	.000	.000	.000
.50	7.756	7.949	8.144	8.342	8.544	.000	.000	.000	.000	.000	.000
.55	7.989	8.165	8.342	8.521	8.702	.000	.000	.000	.000	.000	.000
.60	8.216	8.375	8.533	8.693	8.853	.000	.000	.000	.000	.000	.000
.65	8.438	8.578	8.718	8.858	8.998	.000	.000	.000	.000	.000	.000
.70	8.654	8.776	8.897	9.017	9.137	.000	.000	.000	.000	.000	.000
.75	8.866	8.969	9.070	9.170	9.270	.000	.000	.000	.000	.000	.000
.80	9.074	9.157	9.239	9.319	9.398	.000	.000	.000	.000	.000	.000
.85	9.278	9.341	9.402	9.463	9.522	.000	.000	.000	.000	.000	.000
.90	9.478	9.521	9.562	9.602	9.642	.000	.000	.000	.000	.000	.000
.95	9.675	9.697	9.718	9.738	9.758	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

N=1.0 C/L=0.25

	A/L										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11/12											
.05	5.326	5.680	6.061	6.469	6.903	7.361	.000	.000	.000	.000	.000
.10	5.716	6.060	6.425	6.811	7.217	7.639	.000	.000	.000	.000	.000
.15	6.055	6.388	6.737	7.102	7.481	7.870	.000	.000	.000	.000	.000
.20	6.364	6.683	7.015	7.359	7.712	8.072	.000	.000	.000	.000	.000
.25	6.650	6.955	7.270	7.592	7.921	8.251	.000	.000	.000	.000	.000
.30	6.920	7.210	7.506	7.807	8.111	8.414	.000	.000	.000	.000	.000
.35	7.176	7.449	7.727	8.007	8.287	8.564	.000	.000	.000	.000	.000
.40	7.421	7.677	7.935	8.194	8.450	8.702	.000	.000	.000	.000	.000
.45	7.656	7.895	8.133	8.370	8.604	8.831	.000	.000	.000	.000	.000
.50	7.883	8.104	8.322	8.537	8.748	8.952	.000	.000	.000	.000	.000
.55	8.104	8.305	8.502	8.696	8.884	9.066	.000	.000	.000	.000	.000
.60	8.318	8.499	8.676	8.848	9.014	9.173	.000	.000	.000	.000	.000
.65	8.527	8.687	8.842	8.993	9.137	9.275	.000	.000	.000	.000	.000
.70	8.730	8.869	9.003	9.132	9.255	9.372	.000	.000	.000	.000	.000
.75	8.930	9.046	9.159	9.266	9.368	9.464	.000	.000	.000	.000	.000
.80	9.125	9.219	9.310	9.395	9.476	9.552	.000	.000	.000	.000	.000
.85	9.316	9.387	9.456	9.520	9.580	9.636	.000	.000	.000	.000	.000
.90	9.504	9.552	9.597	9.640	9.680	9.717	.000	.000	.000	.000	.000
.95	9.688	9.712	9.735	9.757	9.777	9.795	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

I1/I2	N=1.0										C/L=0.30	
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
.05	5.571	5.980	6.414	6.875	7.350	7.835	8.304	.000	.000	.000	.000	
.10	5.943	6.340	6.754	7.188	7.628	8.070	8.489	.000	.000	.000	.000	
.15	6.267	6.649	7.044	7.452	7.859	8.264	8.642	.000	.000	.000	.000	
.20	6.561	6.928	7.301	7.684	8.061	8.431	8.774	.000	.000	.000	.000	
.25	6.833	7.183	7.535	7.893	8.241	8.579	8.889	.000	.000	.000	.000	
.30	7.089	7.420	7.752	8.084	8.405	8.713	8.993	.000	.000	.000	.000	
.35	7.332	7.644	7.953	8.261	8.555	8.835	9.087	.000	.000	.000	.000	
.40	7.563	7.856	8.143	8.426	8.694	8.947	9.173	.000	.000	.000	.000	
.45	7.786	8.058	8.322	8.581	8.823	9.051	9.253	.000	.000	.000	.000	
.50	8.001	8.251	8.492	8.727	8.944	9.148	9.327	.000	.000	.000	.000	
.55	8.209	8.437	8.655	8.865	9.059	9.239	9.396	.000	.000	.000	.000	
.60	8.411	8.616	8.810	8.997	9.167	9.324	9.460	.000	.000	.000	.000	
.65	8.608	8.789	8.959	9.122	9.269	9.405	9.521	.000	.000	.000	.000	
.70	8.800	8.956	9.103	9.242	9.367	9.481	9.579	.000	.000	.000	.000	
.75	8.987	9.119	9.241	9.357	9.460	9.554	9.633	.000	.000	.000	.000	
.80	9.170	9.277	9.375	9.467	9.549	9.623	9.685	.000	.000	.000	.000	
.85	9.350	9.431	9.504	9.573	9.634	9.688	9.735	.000	.000	.000	.000	
.90	9.526	9.580	9.630	9.676	9.716	9.751	9.782	.000	.000	.000	.000	
.95	9.699	9.727	9.751	9.774	9.794	9.812	9.827	.000	.000	.000	.000	
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000	.000	.000	

TABLE II (CONTINUED)
N=1.0 C/L=0.35

	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	5.801	6.267	6.760	7.273	7.790	8.288	8.740	9.123	9.457	.000	.000	.000
.10	6.153	6.602	7.071	7.550	8.025	8.474	8.876	9.213	9.502	.000	.000	.000
.15	6.460	6.891	7.335	7.783	8.220	8.628	8.988	9.288	9.543	.000	.000	.000
.20	6.738	7.150	7.569	7.987	8.389	8.760	9.084	9.351	9.581	.000	.000	.000
.25	6.996	7.387	7.782	8.170	8.539	8.876	9.168	9.407	9.616	.000	.000	.000
.30	7.239	7.608	7.977	8.337	8.675	8.981	9.243	9.457	9.616	.000	.000	.000
.35	7.469	7.816	8.159	8.491	8.800	9.076	9.311	9.502	9.648	.000	.000	.000
.40	7.688	8.012	8.330	8.634	8.914	9.163	9.373	9.543	9.679	.000	.000	.000
.45	7.899	8.199	8.491	8.767	9.021	9.243	9.430	9.581	9.707	.000	.000	.000
.50	8.103	8.378	8.644	8.893	9.120	9.318	9.483	9.616	9.734	.000	.000	.000
.55	8.300	8.549	8.789	9.012	9.214	9.388	9.533	9.648	9.760	.000	.000	.000
.60	8.491	8.715	8.928	9.125	9.302	9.454	9.579	9.679	9.760	.000	.000	.000
.65	8.677	8.874	9.061	9.233	9.385	9.515	9.623	9.707	9.760	.000	.000	.000
.70	8.858	9.029	9.189	9.335	9.464	9.574	9.664	9.734	9.760	.000	.000	.000
.75	9.035	9.179	9.312	9.433	9.540	9.629	9.702	9.760	9.760	.000	.000	.000
.80	9.209	9.324	9.431	9.527	9.611	9.682	9.739	9.784	9.807	.000	.000	.000
.85	9.379	9.466	9.546	9.618	9.680	9.732	9.774	9.807	9.829	.000	.000	.000
.90	9.545	9.604	9.657	9.705	9.746	9.780	9.807	9.829	9.850	.000	.000	.000
.95	9.709	9.738	9.765	9.789	9.809	9.826	9.839	9.850	9.870	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000	.000

TABLE II (CONTINUED)

		C/L=0.40										
		A/L										
		N=1.0										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	5.995	6.517	7.064	7.629	8.175	8.683	9.102	9.426	9.639	.000	.000	.000
.10	6.327	6.827	7.343	7.869	8.367	8.823	9.194	9.479	9.666	.000	.000	.000
.15	6.618	7.096	7.582	8.070	8.526	8.938	9.269	9.523	9.688	.000	.000	.000
.20	6.882	7.337	7.793	8.247	8.664	9.038	9.334	9.561	9.708	.000	.000	.000
.25	7.127	7.558	7.985	8.405	8.787	9.125	9.392	9.594	9.725	.000	.000	.000
.30	7.358	7.764	8.162	8.550	8.898	9.204	9.443	9.624	9.740	.000	.000	.000
.35	7.577	7.957	8.326	8.683	8.999	9.275	9.489	9.650	9.754	.000	.000	.000
.40	7.786	8.140	8.481	8.806	9.093	9.341	9.531	9.675	9.767	.000	.000	.000
.45	7.987	8.314	8.626	8.922	9.179	9.401	9.570	9.698	9.779	.000	.000	.000
.50	8.181	8.481	8.764	9.030	9.260	9.457	9.606	9.718	9.790	.000	.000	.000
.55	8.369	8.641	8.895	9.133	9.336	9.510	9.640	9.738	9.800	.000	.000	.000
.60	8.552	8.795	9.021	9.230	9.408	9.559	9.672	9.756	9.810	.000	.000	.000
.65	8.729	8.944	9.141	9.322	9.476	9.605	9.701	9.773	9.819	.000	.000	.000
.70	8.903	9.087	9.256	9.411	9.540	9.649	9.729	9.789	9.827	.000	.000	.000
.75	9.072	9.227	9.367	9.495	9.601	9.690	9.756	9.804	9.835	.000	.000	.000
.80	9.238	9.362	9.474	9.576	9.660	9.730	9.781	9.819	9.843	.000	.000	.000
.85	9.400	9.494	9.578	9.654	9.716	9.767	9.805	9.832	9.850	.000	.000	.000
.90	9.559	9.622	9.678	9.728	9.769	9.803	9.827	9.845	9.857	.000	.000	.000
.95	9.716	9.747	9.775	9.800	9.820	9.837	9.849	9.858	9.863	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000	.000

TABLE II (CONTINUED)
N=1.0 C/L=0.45

I ₁ /I ₂	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
.05	6.127	6.687	7.278	7.876	8.447	8.948	9.342	9.613	9.769	9.841	.000	
.10	6.443	6.977	7.532	8.085	8.604	9.053	9.403	9.642	9.780	9.844	.000	
.15	6.722	7.230	7.751	8.262	8.735	9.141	9.454	9.667	9.790	9.847	.000	
.20	6.976	7.457	7.945	8.418	8.850	9.217	9.498	9.688	9.798	9.849	.000	
.25	7.212	7.667	8.122	8.558	8.953	9.284	9.537	9.707	9.806	9.851	.000	
.30	7.435	7.862	8.285	8.686	9.046	9.345	9.572	9.725	9.812	9.853	.000	
.35	7.646	8.045	8.437	8.804	9.131	9.400	9.604	9.743	9.818	9.855	.000	
.40	7.849	8.219	8.579	8.914	9.209	9.451	9.633	9.754	9.824	9.857	.000	
.45	8.043	8.385	8.714	9.017	9.283	9.499	9.660	9.768	9.829	9.858	.000	
.50	8.231	8.543	8.842	9.115	9.351	9.543	9.685	9.780	9.834	9.859	.000	
.55	8.413	8.696	8.963	9.206	9.416	9.584	9.708	9.791	9.839	9.861	.000	
.60	8.590	8.843	9.080	9.294	9.476	9.623	9.730	9.802	9.843	9.862	.000	
.65	8.763	8.985	9.192	9.377	9.534	9.659	9.751	9.812	9.847	9.863	.000	
.70	8.931	9.122	9.299	9.456	9.589	9.694	9.771	9.822	9.851	9.864	.000	
.75	9.095	9.255	9.402	9.532	9.641	9.727	9.789	9.831	9.854	9.865	.000	
.80	9.256	9.384	9.502	9.605	9.690	9.758	9.807	9.839	9.858	9.866	.000	
.85	9.414	9.510	9.598	9.675	9.738	9.788	9.824	9.847	9.861	9.867	.000	
.90	9.568	9.633	9.691	9.742	9.784	9.816	9.840	9.855	9.864	9.868	.000	
.95	9.720	9.753	9.782	9.807	9.828	9.843	9.855	9.863	9.867	9.869	.000	
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000	

TABLE II (CONTINUED)

G/L=0.50											
N=1.0											
A/L											
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11/12											
.05	6.174	6.749	7.356	7.967	8.545	9.044	9.427	9.683	9.813	9.862	9.870
.10	6.484	7.032	7.600	8.164	8.689	9.136	9.476	9.704	9.819	9.863	9.870
.15	6.758	7.278	7.811	8.332	8.810	9.214	9.518	9.721	9.824	9.864	9.870
.20	7.009	7.501	7.999	8.480	8.916	9.281	9.555	9.737	9.829	9.864	9.870
.25	7.242	7.706	8.170	8.613	9.011	9.341	9.587	9.751	9.833	9.865	9.870
.30	7.461	7.897	8.328	8.735	9.098	9.396	9.617	9.763	9.837	9.865	9.870
.35	7.670	8.077	8.475	8.848	9.177	9.445	9.643	9.775	9.840	9.866	9.870
.40	7.870	8.247	8.614	8.953	9.250	9.491	9.668	9.785	9.843	9.866	9.870
.45	8.063	8.410	8.744	9.052	9.319	9.533	9.691	9.795	9.846	9.867	9.870
.50	8.248	8.566	8.869	9.145	9.383	9.573	9.712	9.804	9.849	9.867	9.870
.55	8.429	8.716	8.987	9.233	9.443	9.610	9.732	9.812	9.852	9.867	9.870
.60	8.604	8.860	9.100	9.316	9.500	9.645	9.751	9.820	9.854	9.868	9.870
.65	8.774	8.999	9.209	9.396	9.554	9.678	9.768	9.827	9.856	9.868	9.870
.70	8.940	9.134	9.313	9.472	9.605	9.710	9.785	9.834	9.859	9.868	9.870
.75	9.103	9.265	9.414	9.545	9.654	9.740	9.801	9.841	9.861	9.869	9.870
.80	9.262	9.392	9.511	9.615	9.701	9.768	9.816	9.847	9.863	9.869	9.870
.85	9.418	9.516	9.605	9.682	9.746	9.795	9.830	9.853	9.865	9.869	9.870
.90	9.571	9.637	9.696	9.747	9.789	9.821	9.844	9.859	9.866	9.869	9.870
.95	9.722	9.755	9.784	9.809	9.830	9.846	9.857	9.864	9.868	9.869	9.870
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870

TABLE II (CONTINUED)

I1/I2	N=2.0											
	C/L=0.00											
	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
.05	2.802	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.10	3.598	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.15	4.212	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.20	4.734	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.25	5.198	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.30	5.622	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.35	6.016	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.386	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.45	6.736	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.070	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.391	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.60	7.699	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.65	7.997	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.286	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.567	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.840	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.106	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.366	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.620	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)
N=2.0 C/L=0.05

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11/12											
.05	2.954	3.126	.000	.000	.000	.000	.000	.000	.000	.000	.000
.10	3.760	3.943	.000	.000	.000	.000	.000	.000	.000	.000	.000
.15	4.375	4.561	.000	.000	.000	.000	.000	.000	.000	.000	.000
.20	4.895	5.078	.000	.000	.000	.000	.000	.000	.000	.000	.000
.25	5.355	5.533	.000	.000	.000	.000	.000	.000	.000	.000	.000
.30	5.773	5.944	.000	.000	.000	.000	.000	.000	.000	.000	.000
.35	6.160	6.322	.000	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.521	6.674	.000	.000	.000	.000	.000	.000	.000	.000	.000
.45	6.863	7.005	.000	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.187	7.318	.000	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.497	7.617	.000	.000	.000	.000	.000	.000	.000	.000	.000
.60	7.795	7.903	.000	.000	.000	.000	.000	.000	.000	.000	.000
.65	8.082	8.178	.000	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.360	8.442	.000	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.628	8.698	.000	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.890	8.946	.000	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.144	9.186	.000	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.391	9.420	.000	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.633	9.648	.000	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

		C/L=0.10										
		A/L										
		N=2.0										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	3.123	3.321	3.545	.000	.000	.000	.000	.000	.000	.000	.000	.000
.10	3.940	4.150	4.386	.000	.000	.000	.000	.000	.000	.000	.000	.000
.15	4.556	4.768	5.004	.000	.000	.000	.000	.000	.000	.000	.000	.000
.20	5.073	5.282	5.513	.000	.000	.000	.000	.000	.000	.000	.000	.000
.25	5.527	5.730	5.953	.000	.000	.000	.000	.000	.000	.000	.000	.000
.30	5.938	6.133	6.345	.000	.000	.000	.000	.000	.000	.000	.000	.000
.35	6.316	6.501	6.702	.000	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.668	6.842	7.031	.000	.000	.000	.000	.000	.000	.000	.000	.000
.45	6.999	7.162	7.337	.000	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.313	7.463	7.624	.000	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.612	7.749	7.895	.000	.000	.000	.000	.000	.000	.000	.000	.000
.60	7.898	8.021	8.153	.000	.000	.000	.000	.000	.000	.000	.000	.000
.65	8.173	8.282	8.398	.000	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.438	8.533	8.633	.000	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.695	8.774	8.858	.000	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.943	9.007	9.074	.000	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.184	9.232	9.283	.000	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.418	9.451	9.485	.000	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.647	9.663	9.680	.000	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)
N=2.0 C/L=0.15

I1/I2	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
.05	3.312	3.542	3.802	4.100	.000	.000	.000	.000	.000	.000	.000	.000
.10	4.137	4.381	4.653	4.959	.000	.000	.000	.000	.000	.000	.000	.000
.15	4.753	4.999	5.270	5.571	.000	.000	.000	.000	.000	.000	.000	.000
.20	5.265	5.507	5.771	6.062	.000	.000	.000	.000	.000	.000	.000	.000
.25	5.712	5.947	6.201	6.478	.000	.000	.000	.000	.000	.000	.000	.000
.30	6.114	6.339	6.581	6.843	.000	.000	.000	.000	.000	.000	.000	.000
.35	6.482	6.696	6.924	7.170	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.824	7.025	7.238	7.466	.000	.000	.000	.000	.000	.000	.000	.000
.45	7.143	7.331	7.528	7.739	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.445	7.618	7.799	7.991	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.732	7.890	8.054	8.227	.000	.000	.000	.000	.000	.000	.000	.000
.60	8.006	8.148	8.294	8.448	.000	.000	.000	.000	.000	.000	.000	.000
.65	8.268	8.393	8.523	8.657	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.520	8.628	8.740	8.855	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.763	8.854	8.947	9.044	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.998	9.071	9.146	9.223	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.225	9.281	9.337	9.395	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.446	9.483	9.521	9.559	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.661	9.679	9.698	9.717	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

		C/L=0.20										
		N=2.0					A/L					
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I1/I2												
.05	3.520	3.790	4.096	4.444	4.844	.000	.000	.000	.000	.000	.000	.000
.10	4.351	4.637	4.954	5.307	5.703	.000	.000	.000	.000	.000	.000	.000
.15	4.963	5.251	5.565	5.909	6.289	.000	.000	.000	.000	.000	.000	.000
.20	5.468	5.751	6.055	6.385	6.744	.000	.000	.000	.000	.000	.000	.000
.25	5.906	6.180	6.471	6.783	7.119	.000	.000	.000	.000	.000	.000	.000
.30	6.297	6.560	6.836	7.129	7.441	.000	.000	.000	.000	.000	.000	.000
.35	6.654	6.903	7.163	7.435	7.723	.000	.000	.000	.000	.000	.000	.000
.40	6.983	7.217	7.459	7.711	7.974	.000	.000	.000	.000	.000	.000	.000
.45	7.290	7.508	7.732	7.963	8.202	.000	.000	.000	.000	.000	.000	.000
.50	7.580	7.780	7.985	8.194	8.410	.000	.000	.000	.000	.000	.000	.000
.55	7.853	8.036	8.221	8.409	8.601	.000	.000	.000	.000	.000	.000	.000
.60	8.114	8.278	8.443	8.610	8.779	.000	.000	.000	.000	.000	.000	.000
.65	8.362	8.508	8.652	8.798	8.945	.000	.000	.000	.000	.000	.000	.000
.70	8.601	8.726	8.851	8.975	9.100	.000	.000	.000	.000	.000	.000	.000
.75	8.831	8.936	9.040	9.143	9.246	.000	.000	.000	.000	.000	.000	.000
.80	9.052	9.137	9.220	9.302	9.384	.000	.000	.000	.000	.000	.000	.000
.85	9.266	9.330	9.393	9.454	9.514	.000	.000	.000	.000	.000	.000	.000
.90	9.473	9.516	9.558	9.599	9.638	.000	.000	.000	.000	.000	.000	.000
.95	9.674	9.696	9.717	9.737	9.757	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)
N=2.0 C/L=0.25

I1/I2	A/L										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.05	3.745	4.066	4.428	4.838	5.307	5.841	.000	.000	.000	.000	.000
.10	4.577	4.915	5.286	5.696	6.149	6.647	.000	.000	.000	.000	.000
.15	5.182	5.520	5.886	6.281	6.707	7.165	.000	.000	.000	.000	.000
.20	5.676	6.008	6.360	6.735	7.132	7.550	.000	.000	.000	.000	.000
.25	6.102	6.422	6.758	7.110	7.478	7.858	.000	.000	.000	.000	.000
.30	6.481	6.787	7.104	7.432	7.770	8.114	.000	.000	.000	.000	.000
.35	6.825	7.114	7.411	7.714	8.023	8.334	.000	.000	.000	.000	.000
.40	7.141	7.412	7.687	7.966	8.247	8.526	.000	.000	.000	.000	.000
.45	7.435	7.687	7.940	8.194	8.448	8.697	.000	.000	.000	.000	.000
.50	7.711	7.942	8.173	8.403	8.629	8.851	.000	.000	.000	.000	.000
.55	7.971	8.181	8.390	8.595	8.796	8.990	.000	.000	.000	.000	.000
.60	8.218	8.406	8.592	8.773	8.949	9.118	.000	.000	.000	.000	.000
.65	8.454	8.620	8.782	8.939	9.091	9.236	.000	.000	.000	.000	.000
.70	8.679	8.822	8.961	9.095	9.223	9.345	.000	.000	.000	.000	.000
.75	8.895	9.015	9.131	9.242	9.347	9.446	.000	.000	.000	.000	.000
.80	9.103	9.200	9.293	9.381	9.464	9.541	.000	.000	.000	.000	.000
.85	9.304	9.377	9.447	9.512	9.573	9.631	.000	.000	.000	.000	.000
.90	9.499	9.547	9.594	9.637	9.677	9.715	.000	.000	.000	.000	.000
.95	9.687	9.711	9.734	9.756	9.776	9.794	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

N=2.0

C/L=0.30

I1/I2	A/L										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.05	3.982	4.365	4.795	5.284	5.831	6.446	7.108	.000	.000	.000	.000
.10	4.807	5.207	5.642	6.121	6.636	7.189	7.753	.000	.000	.000	.000
.15	5.400	5.798	6.222	6.677	7.153	7.649	8.137	.000	.000	.000	.000
.20	5.881	6.269	6.674	7.102	7.538	7.983	8.410	.000	.000	.000	.000
.25	6.293	6.666	7.049	7.448	7.846	8.245	8.621	.000	.000	.000	.000
.30	6.658	7.012	7.372	7.741	8.103	8.461	8.792	.000	.000	.000	.000
.35	6.988	7.321	7.656	7.995	8.324	8.643	8.936	.000	.000	.000	.000
.40	7.290	7.602	7.911	8.221	8.517	8.802	9.059	.000	.000	.000	.000
.45	7.570	7.859	8.142	8.423	8.688	8.941	9.167	.000	.000	.000	.000
.50	7.833	8.097	8.354	8.607	8.843	9.065	9.263	.000	.000	.000	.000
.55	8.080	8.320	8.551	8.775	8.983	9.178	9.348	.000	.000	.000	.000
.60	8.314	8.528	8.733	8.931	9.112	9.280	9.426	.000	.000	.000	.000
.65	8.537	8.725	8.904	9.075	9.230	9.373	9.497	.000	.000	.000	.000
.70	8.750	8.912	9.064	9.209	9.340	9.460	9.563	.000	.000	.000	.000
.75	8.954	9.090	9.216	9.336	9.442	9.540	9.623	.000	.000	.000	.000
.80	9.150	9.259	9.360	9.454	9.538	9.614	9.679	.000	.000	.000	.000
.85	9.339	9.421	9.496	9.566	9.628	9.684	9.731	.000	.000	.000	.000
.90	9.521	9.576	9.626	9.673	9.713	9.750	9.780	.000	.000	.000	.000
.95	9.698	9.726	9.751	9.774	9.793	9.811	9.826	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000	.000	.000

TABLE II (CONTINUED)
N=2.0 C/L=0.35

I1/I2	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
.05	4.219	4.672	5.185	5.762	6.402	7.091	7.796	8.459	9.094	9.626	10.000	1.000
.10	5.028	5.494	6.004	6.556	7.140	7.735	8.309	8.821	9.356	9.822	10.000	.000
.15	5.605	6.063	6.554	7.070	7.599	8.120	8.605	9.025	9.427	9.796	10.000	.000
.20	6.070	6.513	6.978	7.456	7.935	8.394	8.811	9.165	9.487	9.771	10.000	.000
.25	6.466	6.889	7.326	7.767	8.200	8.606	8.968	9.271	9.539	9.755	10.000	.000
.30	6.816	7.216	7.623	8.028	8.418	8.778	9.094	9.356	9.585	9.781	10.000	.000
.35	7.132	7.506	7.883	8.253	8.603	8.923	9.199	9.427	9.622	9.781	10.000	.000
.40	7.421	7.769	8.115	8.451	8.765	9.047	9.289	9.487	9.646	9.781	10.000	.000
.45	7.688	8.009	8.325	8.627	8.907	9.156	9.368	9.539	9.696	9.781	10.000	.000
.50	7.938	8.231	8.517	8.787	9.035	9.253	9.437	9.585	9.727	9.781	10.000	.000
.55	8.174	8.438	8.693	8.933	9.151	9.340	9.499	9.626	9.727	9.781	10.000	.000
.60	8.396	8.632	8.857	9.067	9.256	9.419	9.554	9.662	9.746	9.781	10.000	.000
.65	8.608	8.814	9.010	9.191	9.353	9.491	9.605	9.696	9.766	9.781	10.000	.000
.70	8.810	8.987	9.154	9.307	9.442	9.557	9.652	9.727	9.781	9.781	10.000	.000
.75	9.003	9.151	9.289	9.415	9.525	9.619	9.695	9.755	9.781	9.781	10.000	.000
.80	9.189	9.307	9.417	9.516	9.603	9.676	9.735	9.781	9.781	9.781	10.000	.000
.85	9.368	9.457	9.539	9.612	9.675	9.729	9.772	9.805	9.828	9.828	10.000	.000
.90	9.541	9.600	9.654	9.702	9.744	9.779	9.806	9.828	9.850	9.850	10.000	.000
.95	9.708	9.737	9.764	9.788	9.808	9.825	9.839	9.850	9.870	9.870	10.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	10.000	.000

TABLE II (CONTINUED)

I1/I2	N=2.0										
	C/L=0.40										
	A/L										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.05	4.432	4.963	5.562	6.238	6.967	7.725	8.429	9.021	9.431	.000	.000
.10	5.219	5.754	6.336	6.966	7.609	8.241	8.794	9.241	9.542	.000	.000
.15	5.777	6.297	6.848	7.428	7.999	8.543	9.001	9.364	9.605	.000	.000
.20	6.225	6.723	7.240	7.772	8.281	8.754	9.144	9.448	9.649	.000	.000
.25	6.607	7.079	7.560	8.046	8.501	8.917	9.252	9.512	9.682	.000	.000
.30	6.944	7.387	7.832	8.275	8.682	9.048	9.339	9.563	9.708	.000	.000
.35	7.247	7.661	8.070	8.472	8.835	9.158	9.412	9.605	9.730	.000	.000
.40	7.524	7.908	8.282	8.644	8.968	9.253	9.473	9.641	9.749	.000	.000
.45	7.781	8.133	8.472	8.798	9.085	9.335	9.527	9.672	9.765	.000	.000
.50	8.021	8.341	8.647	8.937	9.190	9.408	9.574	9.700	9.780	.000	.000
.55	8.246	8.535	8.807	9.063	9.284	9.473	9.616	9.724	9.793	.000	.000
.60	8.459	8.716	8.955	9.179	9.370	9.533	9.655	9.746	9.804	.000	.000
.65	8.662	8.886	9.094	9.286	9.449	9.587	9.689	9.766	9.815	.000	.000
.70	8.855	9.048	9.224	9.386	9.522	9.636	9.721	9.785	9.824	.000	.000
.75	9.040	9.201	9.346	9.479	9.590	9.682	9.750	9.801	9.833	.000	.000
.80	9.218	9.346	9.462	9.566	9.653	9.725	9.778	9.817	9.842	.000	.000
.85	9.390	9.485	9.571	9.648	9.712	9.764	9.803	9.832	9.849	.000	.000
.90	9.555	9.619	9.675	9.726	9.767	9.802	9.827	9.845	9.857	.000	.000
.95	9.715	9.747	9.775	9.800	9.820	9.837	9.849	9.858	9.863	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000

TABLE II (CONTINUED)
N=2.0 C/L=0.45

	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	4.587	5.179	5.856	6.612	7.416	8.204	8.887	9.386	9.681	9.817	.000	
.10	5.351	5.936	6.612	7.379	8.185	8.994	9.677	9.506	9.727	9.829	.000	
.15	5.893	6.455	7.057	7.679	8.285	8.994	9.677	9.506	9.727	9.829	.000	
.20	6.328	6.862	7.422	7.986	8.522	9.120	9.677	9.506	9.727	9.829	.000	
.25	6.699	7.202	7.720	8.231	8.708	9.222	9.677	9.506	9.727	9.829	.000	
.30	7.027	7.497	7.973	8.436	8.861	9.307	9.677	9.506	9.727	9.829	.000	
.35	7.321	7.758	8.194	8.612	9.000	9.381	9.677	9.506	9.727	9.829	.000	
.40	7.591	7.994	8.391	8.767	9.103	9.446	9.677	9.506	9.727	9.829	.000	
.45	7.840	8.209	8.569	8.905	9.202	9.503	9.677	9.506	9.727	9.829	.000	
.50	8.073	8.408	8.731	9.029	9.290	9.555	9.677	9.506	9.727	9.829	.000	
.55	8.292	8.593	8.880	9.143	9.370	9.602	9.677	9.506	9.727	9.829	.000	
.60	8.499	8.766	9.019	9.247	9.444	9.644	9.677	9.506	9.727	9.829	.000	
.65	8.696	8.929	9.147	9.344	9.511	9.684	9.677	9.506	9.727	9.829	.000	
.70	8.884	9.083	9.268	9.433	9.573	9.720	9.677	9.506	9.727	9.829	.000	
.75	9.064	9.230	9.382	9.517	9.630	9.754	9.677	9.506	9.727	9.829	.000	
.80	9.237	9.369	9.490	9.596	9.684	9.786	9.677	9.506	9.727	9.829	.000	
.85	9.403	9.502	9.592	9.670	9.735	9.786	9.677	9.506	9.727	9.829	.000	
.90	9.564	9.629	9.689	9.740	9.782	9.815	9.677	9.506	9.727	9.829	.000	
.95	9.719	9.752	9.781	9.806	9.827	9.843	9.677	9.506	9.727	9.829	.000	
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000	

TABLE II (CONTINUED)

N=2.0												
C/L=0.50												
	A/L											
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
I1/I2												
.05	4.645	5.262	5.971	6.761	7.593	8.391	9.057	9.523	9.766	9.856	9.870	
.10	5.399	6.003	6.670	7.379	8.088	8.733	9.251	9.607	9.790	9.859	9.870	
.15	5.934	6.513	7.133	7.772	8.390	8.937	9.364	9.655	9.804	9.861	9.870	
.20	6.365	6.913	7.487	8.065	8.610	9.081	9.444	9.690	9.814	9.862	9.870	
.25	6.732	7.247	7.777	8.299	8.782	9.193	9.506	9.716	9.822	9.863	9.870	
.30	7.055	7.536	8.023	8.495	8.924	9.284	9.556	9.737	9.829	9.864	9.870	
.35	7.347	7.793	8.238	8.663	9.045	9.361	9.598	9.755	9.834	9.865	9.870	
.40	7.614	8.025	8.430	8.811	9.150	9.427	9.633	9.770	9.839	9.866	9.870	
.45	7.860	8.236	8.603	8.944	9.243	9.486	9.665	9.784	9.843	9.866	9.870	
.50	8.091	8.432	8.760	9.063	9.326	9.537	9.693	9.796	9.847	9.867	9.870	
.55	8.308	8.614	8.906	9.172	9.401	9.584	9.718	9.806	9.850	9.867	9.870	
.60	8.513	8.784	9.040	9.272	9.469	9.626	9.740	9.816	9.853	9.867	9.870	
.65	8.708	8.944	9.166	9.364	9.532	9.665	9.761	9.824	9.855	9.868	9.870	
.70	8.894	9.096	9.284	9.450	9.590	9.701	9.780	9.832	9.858	9.868	9.870	
.75	9.072	9.240	9.395	9.531	9.645	9.734	9.798	9.840	9.860	9.869	9.870	
.80	9.243	9.377	9.499	9.606	9.695	9.764	9.814	9.847	9.862	9.869	9.870	
.85	9.408	9.508	9.599	9.678	9.743	9.793	9.829	9.853	9.864	9.869	9.870	
.90	9.567	9.633	9.693	9.745	9.787	9.820	9.844	9.859	9.866	9.869	9.870	
.95	9.721	9.754	9.783	9.809	9.830	9.846	9.857	9.864	9.868	9.869	9.870	
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	

TABLE II (CONTINUED)

N=3.0		C/L=0.00										
		A/L										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	2.391	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.10	3.273	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.15	3.948	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.20	4.518	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.25	5.021	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.30	5.477	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.35	5.897	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.290	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.45	6.659	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.009	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.343	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.60	7.663	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.65	7.971	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.267	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.554	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.832	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.102	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.364	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.620	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)
N=3.0 C/L=0.05

	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	2.524	2.676	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.10	3.424	3.596	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.15	4.105	4.283	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.20	4.675	4.854	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.25	5.176	5.350	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.30	5.626	5.795	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.35	6.040	6.201	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.425	6.577	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.45	6.785	6.928	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.126	7.258	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.450	7.570	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.60	7.759	7.867	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.65	8.056	8.151	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.341	8.424	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.616	8.686	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.882	8.938	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.139	9.182	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.389	9.418	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.633	9.647	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

I1/I2	N=3.0										
	G/L=0.10										
	A/L										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.05	2.674	2.850	3.052	.000	.000	.000	.000	.000	.000	.000	.000
.10	3.593	3.791	4.015	.000	.000	.000	.000	.000	.000	.000	.000
.15	4.279	4.484	4.714	.000	.000	.000	.000	.000	.000	.000	.000
.20	4.849	5.054	5.282	.000	.000	.000	.000	.000	.000	.000	.000
.25	5.345	5.546	5.768	.000	.000	.000	.000	.000	.000	.000	.000
.30	5.790	5.984	6.197	.000	.000	.000	.000	.000	.000	.000	.000
.35	6.196	6.381	6.583	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.571	6.746	6.936	.000	.000	.000	.000	.000	.000	.000	.000
.45	6.922	7.085	7.262	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.252	7.403	7.565	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.565	7.702	7.850	.000	.000	.000	.000	.000	.000	.000	.000
.60	7.863	7.986	8.119	.000	.000	.000	.000	.000	.000	.000	.000
.65	8.147	8.256	8.373	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.420	8.514	8.615	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.682	8.762	8.846	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.935	8.999	9.067	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.180	9.228	9.279	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.417	9.449	9.483	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.646	9.663	9.680	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

		N=3.0										C/L=0.15									
		A/L																			
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0									
11/12																					
.05		2.843	3.049	3.285	3.557	.000	.000	.000	.000	.000	.000	.000									
.10		3.779	4.011	4.271	4.567	.000	.000	.000	.000	.000	.000	.000									
.15		4.470	4.709	4.975	5.273	.000	.000	.000	.000	.000	.000	.000									
.20		5.037	5.277	5.539	5.830	.000	.000	.000	.000	.000	.000	.000									
.25		5.528	5.762	6.016	6.296	.000	.000	.000	.000	.000	.000	.000									
.30		5.965	6.191	6.434	6.699	.000	.000	.000	.000	.000	.000	.000									
.35		6.362	6.577	6.807	7.056	.000	.000	.000	.000	.000	.000	.000									
.40		6.727	6.930	7.145	7.376	.000	.000	.000	.000	.000	.000	.000									
.45		7.066	7.256	7.455	7.668	.000	.000	.000	.000	.000	.000	.000									
.50		7.385	7.560	7.743	7.937	.000	.000	.000	.000	.000	.000	.000									
.55		7.685	7.845	8.011	8.186	.000	.000	.000	.000	.000	.000	.000									
.60		7.970	8.114	8.262	8.418	.000	.000	.000	.000	.000	.000	.000									
.65		8.242	8.368	8.499	8.635	.000	.000	.000	.000	.000	.000	.000									
.70		8.502	8.611	8.723	8.840	.000	.000	.000	.000	.000	.000	.000									
.75		8.751	8.843	8.936	9.033	.000	.000	.000	.000	.000	.000	.000									
.80		8.990	9.064	9.139	9.217	.000	.000	.000	.000	.000	.000	.000									
.85		9.221	9.277	9.333	9.391	.000	.000	.000	.000	.000	.000	.000									
.90		9.444	9.482	9.519	9.558	.000	.000	.000	.000	.000	.000	.000									
.95		9.660	9.679	9.698	9.717	.000	.000	.000	.000	.000	.000	.000									
1.00		9.870	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000	.000									

TABLE II (CONTINUED)

I ¹ /I ²	N=3.0										
	G/L=0.20										
	A/L										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.05	3.029	3.274	3.554	3.876	4.253	.000	.000	.000	.000	.000	.000
.10	3.982	4.256	4.562	4.907	5.299	.000	.000	.000	.000	.000	.000
.15	4.674	4.957	5.267	5.610	5.993	.000	.000	.000	.000	.000	.000
.20	5.237	5.519	5.823	6.155	6.520	.000	.000	.000	.000	.000	.000
.25	5.720	5.995	6.289	6.605	6.948	.000	.000	.000	.000	.000	.000
.30	6.148	6.412	6.692	6.989	7.308	.000	.000	.000	.000	.000	.000
.35	6.534	6.785	7.048	7.326	7.620	.000	.000	.000	.000	.000	.000
.40	6.887	7.124	7.369	7.626	7.895	.000	.000	.000	.000	.000	.000
.45	7.214	7.435	7.662	7.897	8.141	.000	.000	.000	.000	.000	.000
.50	7.520	7.723	7.931	8.144	8.364	.000	.000	.000	.000	.000	.000
.55	7.807	7.992	8.180	8.371	8.567	.000	.000	.000	.000	.000	.000
.60	8.079	8.245	8.412	8.582	8.753	.000	.000	.000	.000	.000	.000
.65	8.337	8.484	8.630	8.778	8.926	.000	.000	.000	.000	.000	.000
.70	8.583	8.710	8.836	8.961	9.087	.000	.000	.000	.000	.000	.000
.75	8.819	8.925	9.030	9.134	9.238	.000	.000	.000	.000	.000	.000
.80	9.045	9.130	9.214	9.297	9.379	.000	.000	.000	.000	.000	.000
.85	9.262	9.326	9.389	9.451	9.512	.000	.000	.000	.000	.000	.000
.90	9.471	9.514	9.556	9.597	9.637	.000	.000	.000	.000	.000	.000
.95	9.674	9.695	9.716	9.737	9.756	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)
N=3.0 C/L=0.25

	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	3.234	3.528	3.862	4.248	4.697	5.222	5.222	.000	.000	.000	.000	.000
.10	4.198	4.525	4.887	5.292	5.747	6.257	6.257	.000	.000	.000	.000	.000
.15	4.888	5.223	5.587	5.985	6.420	6.894	6.894	.000	.000	.000	.000	.000
.20	5.444	5.776	6.131	6.511	6.920	7.354	7.354	.000	.000	.000	.000	.000
.25	5.917	6.239	6.579	6.939	7.317	7.712	7.712	.000	.000	.000	.000	.000
.30	6.332	6.641	6.964	7.299	7.647	8.005	8.005	.000	.000	.000	.000	.000
.35	6.706	6.999	7.301	7.611	7.929	8.251	8.251	.000	.000	.000	.000	.000
.40	7.046	7.321	7.602	7.887	8.175	8.464	8.464	.000	.000	.000	.000	.000
.45	7.360	7.615	7.874	8.133	8.393	8.650	8.650	.000	.000	.000	.000	.000
.50	7.652	7.887	8.122	8.356	8.588	8.816	8.816	.000	.000	.000	.000	.000
.55	7.926	8.139	8.351	8.560	8.765	8.964	8.964	.000	.000	.000	.000	.000
.60	8.184	8.375	8.563	8.747	8.927	9.099	9.099	.000	.000	.000	.000	.000
.65	8.429	8.597	8.761	8.921	9.075	9.222	9.222	.000	.000	.000	.000	.000
.70	8.662	8.806	8.947	9.083	9.212	9.336	9.336	.000	.000	.000	.000	.000
.75	8.884	9.005	9.122	9.234	9.340	9.441	9.441	.000	.000	.000	.000	.000
.80	9.096	9.194	9.287	9.376	9.459	9.538	9.538	.000	.000	.000	.000	.000
.85	9.301	9.374	9.443	9.509	9.571	9.629	9.629	.000	.000	.000	.000	.000
.90	9.497	9.546	9.592	9.636	9.676	9.714	9.714	.000	.000	.000	.000	.000
.95	9.687	9.711	9.734	9.756	9.776	9.794	9.794	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

C/L=0.30												
N=3.0												
I1/I2	A/L											
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
.05	3.453	3.807	4.210	4.677	5.214	5.837	6.539	.000	.000	.000	.000	
.10	4.421	4.811	5.240	5.720	6.246	6.824	7.432	.000	.000	.000	.000	
.15	5.104	5.500	5.926	6.390	6.882	7.404	7.929	.000	.000	.000	.000	
.20	5.648	6.039	6.450	6.889	7.342	7.810	8.266	.000	.000	.000	.000	
.25	6.108	6.486	6.877	7.287	7.700	8.119	8.518	.000	.000	.000	.000	
.30	6.510	6.870	7.238	7.618	7.993	8.367	8.717	.000	.000	.000	.000	
.35	6.870	7.210	7.552	7.901	8.241	8.574	8.880	.000	.000	.000	.000	
.40	7.196	7.514	7.830	8.148	8.454	8.749	9.018	.000	.000	.000	.000	
.45	7.497	7.791	8.080	8.368	8.641	8.902	9.136	.000	.000	.000	.000	
.50	7.775	8.045	8.307	8.565	8.807	9.036	9.240	.000	.000	.000	.000	
.55	8.036	8.280	8.515	8.744	8.957	9.156	9.332	.000	.000	.000	.000	
.60	8.281	8.499	8.707	8.908	9.093	9.264	9.414	.000	.000	.000	.000	
.65	8.513	8.704	8.885	9.059	9.217	9.363	9.489	.000	.000	.000	.000	
.70	8.733	8.897	9.051	9.198	9.331	9.452	9.557	.000	.000	.000	.000	
.75	8.942	9.080	9.207	9.328	9.436	9.535	9.619	.000	.000	.000	.000	
.80	9.143	9.253	9.355	9.450	9.535	9.612	9.677	.000	.000	.000	.000	
.85	9.335	9.418	9.493	9.564	9.626	9.683	9.730	.000	.000	.000	.000	
.90	9.520	9.575	9.625	9.672	9.712	9.749	9.780	.000	.000	.000	.000	
.95	9.698	9.725	9.750	9.773	9.793	9.811	9.826	.000	.000	.000	.000	
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000	.000	.000	

TABLE II (CONTINUED)
N=3.0 C/L=0.35

	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	3.675	4.100	4.588	5.151	5.797	6.524	7.307	8.088	8.863	9.634	1.000	.000
.10	4.638	5.096	5.606	6.167	6.776	7.414	8.049	8.634	9.218	9.799	1.000	.000
.15	5.307	5.768	6.266	6.798	7.354	7.912	8.441	8.910	9.388	9.866	1.000	.000
.20	5.837	6.286	6.762	7.258	7.760	8.250	8.700	9.108	9.573	1.000	1.000	.000
.25	6.282	6.713	7.161	7.619	8.072	8.502	8.890	9.218	9.573	9.925	1.000	.000
.30	6.670	7.078	7.496	7.915	8.323	8.702	9.038	9.318	9.573	9.825	1.000	.000
.35	7.016	7.398	7.785	8.167	8.532	8.867	9.158	9.399	9.573	9.799	1.000	.000
.40	7.329	7.684	8.040	8.385	8.711	9.006	9.259	9.466	9.634	9.799	1.000	.000
.45	7.616	7.944	8.267	8.578	8.867	9.125	9.345	9.523	9.692	9.866	1.000	.000
.50	7.882	8.181	8.473	8.750	9.005	9.230	9.420	9.573	9.724	9.866	1.000	.000
.55	8.130	8.400	8.660	8.906	9.128	9.323	9.487	9.617	9.753	9.888	1.000	.000
.60	8.364	8.604	8.833	9.047	9.240	9.407	9.546	9.657	9.753	9.825	1.000	.000
.65	8.584	8.794	8.993	9.177	9.341	9.483	9.599	9.692	9.753	9.825	1.000	.000
.70	8.793	8.973	9.142	9.297	9.434	9.552	9.648	9.724	9.799	9.866	1.000	.000
.75	8.992	9.142	9.282	9.409	9.520	9.615	9.692	9.753	9.825	9.888	1.000	.000
.80	9.182	9.302	9.413	9.513	9.600	9.673	9.733	9.780	9.825	9.866	1.000	.000
.85	9.364	9.454	9.536	9.610	9.674	9.728	9.771	9.805	9.825	9.866	1.000	.000
.90	9.539	9.599	9.653	9.702	9.743	9.778	9.806	9.825	9.849	9.866	1.000	.000
.95	9.707	9.737	9.764	9.788	9.808	9.825	9.839	9.849	9.866	9.888	1.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	1.000	.000

TABLE II (CONTINUED)

		C/L=0.40										
		A/L										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I1/I2												
.05	3.879	4.384	4.966	5.643	6.403	7.235	8.056	8.785	9.308	.000	.000	.000
.10	4.827	5.359	5.948	6.600	7.284	7.978	8.605	9.125	9.481	.000	.000	.000
.15	5.480	6.006	6.572	7.177	7.785	8.376	8.884	9.293	9.568	.000	.000	.000
.20	5.994	6.502	7.035	7.591	8.131	8.640	9.065	9.402	9.624	.000	.000	.000
.25	6.425	6.908	7.405	7.912	8.392	8.836	9.197	9.480	9.665	.000	.000	.000
.30	6.799	7.254	7.713	8.174	8.602	8.989	9.300	9.540	9.696	.000	.000	.000
.35	7.132	7.557	7.978	8.395	8.775	9.115	9.383	9.588	9.721	.000	.000	.000
.40	7.433	7.826	8.211	8.586	8.923	9.220	9.452	9.629	9.742	.000	.000	.000
.45	7.710	8.070	8.419	8.754	9.051	9.311	9.511	9.663	9.760	.000	.000	.000
.50	7.965	8.293	8.606	8.904	9.165	9.390	9.563	9.693	9.776	.000	.000	.000
.55	8.204	8.498	8.776	9.039	9.266	9.461	9.608	9.719	9.790	.000	.000	.000
.60	8.428	8.689	8.933	9.161	9.357	9.523	9.649	9.743	9.802	.000	.000	.000
.65	8.639	8.867	9.078	9.274	9.440	9.580	9.685	9.764	9.814	.000	.000	.000
.70	8.839	9.034	9.213	9.377	9.516	9.632	9.718	9.783	9.824	.000	.000	.000
.75	9.030	9.192	9.339	9.473	9.585	9.679	9.749	9.800	9.833	.000	.000	.000
.80	9.212	9.341	9.457	9.563	9.650	9.723	9.777	9.816	9.841	.000	.000	.000
.85	9.386	9.483	9.569	9.647	9.711	9.764	9.802	9.831	9.849	.000	.000	.000
.90	9.553	9.617	9.674	9.725	9.767	9.801	9.826	9.845	9.857	.000	.000	.000
.95	9.714	9.746	9.774	9.800	9.820	9.837	9.849	9.858	9.863	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000	.000

TABLE II (CONTINUED)
N=3.0 C/L=0.45

	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	4.031	4.601	5.273	6.050	6.915	7.812	8.631	9.381	9.254	9.629	9.803	.000
.10	4.959	5.546	6.205	6.925	7.674	8.390	9.000	9.500	9.440	9.701	9.822	.000
.15	5.997	6.169	6.792	7.447	8.100	8.698	9.188	9.588	9.535	9.737	9.832	.000
.20	6.099	6.646	7.226	7.819	8.393	8.903	9.312	9.640	9.596	9.761	9.839	.000
.25	6.519	7.035	7.572	8.108	8.614	9.055	9.402	9.640	9.640	9.779	9.844	.000
.30	6.883	7.367	7.860	8.344	8.791	9.175	9.472	9.675	9.675	9.793	9.848	.000
.35	7.208	7.656	8.108	8.543	8.939	9.273	9.529	9.703	9.703	9.804	9.851	.000
.40	7.501	7.915	8.325	8.714	9.064	9.356	9.577	9.727	9.727	9.813	9.853	.000
.45	7.770	8.148	8.518	8.865	9.173	9.427	9.618	9.747	9.747	9.821	9.856	.000
.50	8.018	8.361	8.692	9.009	9.269	9.489	9.654	9.765	9.765	9.828	9.858	.000
.55	8.250	8.557	8.851	9.121	9.355	9.544	9.686	9.780	9.780	9.834	9.859	.000
.60	8.468	8.740	8.997	9.231	9.432	9.594	9.714	9.794	9.794	9.840	9.861	.000
.65	8.673	8.910	9.132	9.332	9.503	9.639	9.740	9.806	9.806	9.845	9.862	.000
.70	8.868	9.070	9.258	9.425	9.567	9.680	9.763	9.818	9.818	9.849	9.864	.000
.75	9.053	9.221	9.375	9.512	9.627	9.718	9.784	9.828	9.828	9.853	9.865	.000
.80	9.230	9.364	9.486	9.593	9.682	9.753	9.804	9.838	9.838	9.857	9.866	.000
.85	9.400	9.499	9.590	9.668	9.734	9.785	9.822	9.847	9.847	9.861	9.867	.000
.90	9.562	9.628	9.688	9.739	9.782	9.815	9.839	9.855	9.855	9.864	9.868	.000
.95	9.719	9.752	9.781	9.806	9.827	9.843	9.855	9.862	9.862	9.867	9.869	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000

TABLE II (CONTINUED)

		N=3.0										
		C/L=0.50										
		A/L										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I1/I2												
.05	4.088	4.687	5.397	6.219	7.128	8.047	8.851	9.432	9.739	9.852	9.870	
.10	5.007	5.616	6.303	7.052	7.824	8.548	9.144	9.560	9.776	9.857	9.870	
.15	5.639	6.230	6.874	7.550	8.217	8.819	9.297	9.627	9.795	9.860	9.870	
.20	6.136	6.698	7.296	7.905	8.488	9.000	9.399	9.670	9.808	9.861	9.870	
.25	6.552	7.081	7.632	8.181	8.694	9.135	9.474	9.702	9.818	9.863	9.870	
.30	6.913	7.407	7.912	8.406	8.859	9.242	9.532	9.728	9.826	9.864	9.870	
.35	7.234	7.692	8.153	8.596	8.996	9.330	9.580	9.748	9.832	9.865	9.870	
.40	7.524	7.946	8.364	8.760	9.113	9.404	9.621	9.765	9.837	9.865	9.870	
.45	7.790	8.176	8.553	8.905	9.215	9.468	9.655	9.780	9.842	9.866	9.870	
.50	8.037	8.385	8.723	9.034	9.305	9.525	9.686	9.793	9.846	9.866	9.870	
.55	8.266	8.579	8.877	9.150	9.386	9.575	9.713	9.804	9.849	9.867	9.870	
.60	8.482	8.758	9.020	9.256	9.458	9.619	9.737	9.814	9.852	9.867	9.870	
.65	8.685	8.926	9.151	9.353	9.525	9.660	9.758	9.823	9.855	9.868	9.870	
.70	8.878	9.083	9.273	9.443	9.585	9.697	9.778	9.832	9.858	9.868	9.870	
.75	9.062	9.231	9.388	9.526	9.641	9.732	9.797	9.839	9.860	9.868	9.870	
.80	9.237	9.372	9.495	9.603	9.693	9.763	9.813	9.846	9.862	9.869	9.870	
.85	9.404	9.505	9.597	9.676	9.742	9.793	9.829	9.853	9.864	9.869	9.870	
.90	9.565	9.632	9.692	9.744	9.787	9.820	9.843	9.859	9.866	9.869	9.870	
.95	9.720	9.754	9.783	9.809	9.830	9.846	9.857	9.864	9.868	9.869	9.870	
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	

TABLE II (CONTINUED)

		N=4.0										C/L=0.00	
		A/L											
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
11/12													
.05	2.207	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.10	3.121	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.15	3.822	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.20	4.414	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.25	4.935	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.30	5.406	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.35	5.839	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.242	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.45	6.621	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.50	6.979	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.319	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.60	7.645	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.65	7.957	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.258	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.547	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.828	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.099	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.363	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.620	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

		C/L=0.05										
		N=4.0										
		A/L										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	2.331		2.473	.000	.000	.000	.000	.000	.000	.000	.000	.000
.10	3.266		3.432	.000	.000	.000	.000	.000	.000	.000	.000	.000
.15	3.975		4.150	.000	.000	.000	.000	.000	.000	.000	.000	.000
.20	4.568		4.744	.000	.000	.000	.000	.000	.000	.000	.000	.000
.25	5.087		5.261	.000	.000	.000	.000	.000	.000	.000	.000	.000
.30	5.554		5.722	.000	.000	.000	.000	.000	.000	.000	.000	.000
.35	5.981		6.142	.000	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.377		6.529	.000	.000	.000	.000	.000	.000	.000	.000	.000
.45	6.747		6.889	.000	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.096		7.227	.000	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.426		7.547	.000	.000	.000	.000	.000	.000	.000	.000	.000
.60	7.741		7.850	.000	.000	.000	.000	.000	.000	.000	.000	.000
.65	8.042		8.138	.000	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.331		8.414	.000	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.609		8.679	.000	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.878		8.934	.000	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.137		9.180	.000	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.389		9.417	.000	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.633		9.647	.000	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870		9.870	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

		C/L=0.10										
		N=4.0										
		A/L										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	2.471	2.636	2.825	.000	.000	.000	.000	.000	.000	.000	.000	.000
.10	3.428	3.620	3.838	.000	.000	.000	.000	.000	.000	.000	.000	.000
.15	4.146	4.347	4.573	.000	.000	.000	.000	.000	.000	.000	.000	.000
.20	4.740	4.942	5.169	.000	.000	.000	.000	.000	.000	.000	.000	.000
.25	5.256	5.455	5.676	.000	.000	.000	.000	.000	.000	.000	.000	.000
.30	5.717	5.910	6.123	.000	.000	.000	.000	.000	.000	.000	.000	.000
.35	6.136	6.321	6.524	.000	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.523	6.698	6.888	.000	.000	.000	.000	.000	.000	.000	.000	.000
.45	6.884	7.047	7.224	.000	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.222	7.373	7.536	.000	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.541	7.679	7.828	.000	.000	.000	.000	.000	.000	.000	.000	.000
.60	7.845	7.969	8.102	.000	.000	.000	.000	.000	.000	.000	.000	.000
.65	8.134	8.244	8.361	.000	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.410	8.505	8.606	.000	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.676	8.756	8.840	.000	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.931	8.996	9.064	.000	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.178	9.226	9.277	.000	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.416	9.448	9.482	.000	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.646	9.662	9.680	.000	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)
N=4.0 C/L=0.15

	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	2.628	2.822	3.044	3.303	.000	.000	.000	.000	.000	.000	.000	.000
.10	3.608	3.834	4.087	4.376	.000	.000	.000	.000	.000	.000	.000	.000
.15	4.332	4.568	4.830	5.125	.000	.000	.000	.000	.000	.000	.000	.000
.20	4.926	5.163	5.424	5.714	.000	.000	.000	.000	.000	.000	.000	.000
.25	5.437	5.670	5.924	6.204	.000	.000	.000	.000	.000	.000	.000	.000
.30	5.891	6.117	6.360	6.626	.000	.000	.000	.000	.000	.000	.000	.000
.35	6.302	6.517	6.748	6.998	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.679	6.882	7.098	7.331	.000	.000	.000	.000	.000	.000	.000	.000
.45	7.028	7.218	7.419	7.633	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.355	7.530	7.714	7.910	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.662	7.822	7.989	8.165	.000	.000	.000	.000	.000	.000	.000	.000
.60	7.953	8.097	8.246	8.402	.000	.000	.000	.000	.000	.000	.000	.000
.65	8.229	8.356	8.487	8.624	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.492	8.602	8.715	8.832	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.744	8.837	8.931	9.028	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.986	9.061	9.136	9.214	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.219	9.275	9.332	9.390	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.443	9.481	9.519	9.557	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.660	9.679	9.698	9.717	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

N=4.0 $C/L=0.20$

11/12	A/L										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.05	2.804	3.035	3.299	3.607	3.968	.000	.000	.000	.000	.000	.000
.10	3.805	4.072	4.371	4.710	5.098	.000	.000	.000	.000	.000	.000
.15	4.533	4.812	5.119	5.461	5.844	.000	.000	.000	.000	.000	.000
.20	5.124	5.404	5.707	6.040	6.407	.000	.000	.000	.000	.000	.000
.25	5.628	5.903	6.197	6.515	6.861	.000	.000	.000	.000	.000	.000
.30	6.073	6.338	6.619	6.919	7.241	.000	.000	.000	.000	.000	.000
.35	6.474	6.726	6.991	7.271	7.568	.000	.000	.000	.000	.000	.000
.40	6.839	7.077	7.324	7.583	7.855	.000	.000	.000	.000	.000	.000
.45	7.176	7.398	7.626	7.863	8.110	.000	.000	.000	.000	.000	.000
.50	7.490	7.694	7.903	8.118	8.340	.000	.000	.000	.000	.000	.000
.55	7.784	7.970	8.159	8.352	8.549	.000	.000	.000	.000	.000	.000
.60	8.062	8.229	8.397	8.567	8.741	.000	.000	.000	.000	.000	.000
.65	8.324	8.472	8.619	8.767	8.917	.000	.000	.000	.000	.000	.000
.70	8.574	8.701	8.828	8.954	9.081	.000	.000	.000	.000	.000	.000
.75	8.813	8.919	9.025	9.129	9.234	.000	.000	.000	.000	.000	.000
.80	9.041	9.127	9.211	9.294	9.376	.000	.000	.000	.000	.000	.000
.85	9.260	9.324	9.388	9.449	9.510	.000	.000	.000	.000	.000	.000
.90	9.471	9.514	9.556	9.597	9.637	.000	.000	.000	.000	.000	.000
.95	9.674	9.695	9.716	9.737	9.756	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)
N=4.0 C/L=0.25

	0.0	0.1	0.2	0.3	0.4	A/L	0.5	0.6	0.7	0.8	0.9	1.0
I1/I2												
.05	2.997	3.275	3.593	3.963	4.397	4.911	.000	.000	.000	.000	.000	.000
.10	4.015	4.335	4.690	5.091	5.543	6.056	.000	.000	.000	.000	.000	.000
.15	4.744	5.076	5.438	5.836	6.274	6.754	.000	.000	.000	.000	.000	.000
.20	5.329	5.660	6.015	6.398	6.811	7.253	.000	.000	.000	.000	.000	.000
.25	5.824	6.147	6.489	6.852	7.235	7.637	.000	.000	.000	.000	.000	.000
.30	6.258	6.568	6.893	7.232	7.585	7.949	.000	.000	.000	.000	.000	.000
.35	6.646	6.941	7.245	7.559	7.881	8.209	.000	.000	.000	.000	.000	.000
.40	6.998	7.275	7.558	7.847	8.139	8.432	.000	.000	.000	.000	.000	.000
.45	7.322	7.580	7.840	8.102	8.365	8.626	.000	.000	.000	.000	.000	.000
.50	7.623	7.859	8.096	8.333	8.567	8.798	.000	.000	.000	.000	.000	.000
.55	7.903	8.118	8.331	8.542	8.750	8.951	.000	.000	.000	.000	.000	.000
.60	8.167	8.359	8.549	8.735	8.915	9.090	.000	.000	.000	.000	.000	.000
.65	8.416	8.585	8.751	8.912	9.067	9.216	.000	.000	.000	.000	.000	.000
.70	8.653	8.798	8.940	9.076	9.207	9.331	.000	.000	.000	.000	.000	.000
.75	8.878	9.000	9.117	9.230	9.337	9.438	.000	.000	.000	.000	.000	.000
.80	9.093	9.190	9.284	9.373	9.457	9.536	.000	.000	.000	.000	.000	.000
.85	9.299	9.372	9.442	9.508	9.570	9.628	.000	.000	.000	.000	.000	.000
.90	9.496	9.545	9.592	9.635	9.676	9.714	.000	.000	.000	.000	.000	.000
.95	9.686	9.711	9.734	9.756	9.776	9.794	.000	.000	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000	.000	.000	.000	.000

TABLE II (CONTINUED)

		C/L=0.30										
		N=4.0										
		A/L										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	3.204	3.541	3.928	4.378	4.904	5.522	6.233		.000	.000	.000	.000
.10	4.233	4.616	5.040	5.517	6.045	6.633	7.260		.000	.000	.000	.000
.15	4.957	5.352	5.778	6.244	6.742	7.276	7.819		.000	.000	.000	.000
.20	5.532	5.923	6.337	6.780	7.241	7.720	8.191		.000	.000	.000	.000
.25	6.015	6.395	6.789	7.204	7.625	8.054	8.464		.000	.000	.000	.000
.30	6.436	6.799	7.170	7.555	7.937	8.319	8.678		.000	.000	.000	.000
.35	6.810	7.153	7.499	7.853	8.198	8.538	8.851		.000	.000	.000	.000
.40	7.149	7.470	7.789	8.111	8.422	8.723	8.996		.000	.000	.000	.000
.45	7.459	7.756	8.048	8.340	8.617	8.882	9.120		.000	.000	.000	.000
.50	7.746	8.018	8.283	8.544	8.789	9.022	9.228		.000	.000	.000	.000
.55	8.014	8.260	8.497	8.729	8.943	9.146	9.324		.000	.000	.000	.000
.60	8.264	8.484	8.693	8.896	9.083	9.257	9.408		.000	.000	.000	.000
.65	8.501	8.693	8.875	9.051	9.210	9.357	9.485		.000	.000	.000	.000
.70	8.724	8.890	9.045	9.193	9.326	9.449	9.554		.000	.000	.000	.000
.75	8.937	9.075	9.203	9.325	9.433	9.533	9.618		.000	.000	.000	.000
.80	9.139	9.250	9.352	9.448	9.533	9.610	9.676		.000	.000	.000	.000
.85	9.333	9.416	9.492	9.563	9.625	9.682	9.730		.000	.000	.000	.000
.90	9.519	9.574	9.625	9.671	9.712	9.749	9.779		.000	.000	.000	.000
.95	9.698	9.725	9.750	9.773	9.793	9.811	9.826		.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870		.000	.000	.000	.000

TABLE II (CONTINUED)

		C/L=0.35										
		A/L										
N=4.0		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05	3.417	3.823	4.295	4.844	5.484	6.218	7.031	7.869	.000	.000	.000	.000
.10	4.446	4.899	5.405	5.968	6.585	7.242	7.906	8.529	.000	.000	.000	.000
.15	5.159	5.619	6.120	6.658	7.225	7.800	8.353	8.847	.000	.000	.000	.000
.20	5.721	6.172	6.652	7.156	7.669	8.173	8.641	9.047	.000	.000	.000	.000
.25	6.190	6.624	7.077	7.542	8.005	8.448	8.848	9.189	.000	.000	.000	.000
.30	6.596	7.008	7.432	7.858	8.274	8.663	9.008	9.297	.000	.000	.000	.000
.35	6.957	7.344	7.735	8.123	8.495	8.838	9.136	9.384	.000	.000	.000	.000
.40	7.282	7.642	8.001	8.352	8.684	8.984	9.243	9.455	.000	.000	.000	.000
.45	7.580	7.910	8.238	8.553	8.847	9.109	9.333	9.516	.000	.000	.000	.000
.50	7.854	8.155	8.450	8.731	8.990	9.219	9.412	9.568	.000	.000	.000	.000
.55	8.109	8.381	8.644	8.892	9.117	9.315	9.480	9.613	.000	.000	.000	.000
.60	8.348	8.589	8.821	9.037	9.232	9.401	9.541	9.654	.000	.000	.000	.000
.65	8.572	8.784	8.984	9.170	9.336	9.478	9.596	9.690	.000	.000	.000	.000
.70	8.785	8.966	9.136	9.292	9.431	9.549	9.646	9.723	.000	.000	.000	.000
.75	8.987	9.137	9.278	9.405	9.518	9.613	9.691	9.752	.000	.000	.000	.000
.80	9.179	9.299	9.410	9.511	9.598	9.672	9.732	9.780	.000	.000	.000	.000
.85	9.362	9.452	9.535	9.609	9.673	9.727	9.771	9.805	.000	.000	.000	.000
.90	9.538	9.598	9.653	9.701	9.743	9.778	9.806	9.828	.000	.000	.000	.000
.95	9.707	9.737	9.764	9.788	9.808	9.825	9.839	9.849	.000	.000	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000	.000	.000

TABLE II (CONTINUED)

I1/I2	N=4.0										
	C/L=0.40										
	A/L										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.05	3.614	4.100	4.668	5.337	6.102	6.961	7.837	8.641	9.232	.000	.000
.10	4.633	5.161	5.750	6.410	7.110	7.834	8.498	9.059	9.447	.000	.000
.15	5.331	5.859	6.430	7.047	7.671	8.286	8.820	9.255	9.548	.000	.000
.20	5.878	6.390	6.930	7.497	8.052	8.580	9.023	9.376	9.611	.000	.000
.25	6.333	6.821	7.325	7.843	8.335	8.793	9.168	9.462	9.655	.000	.000
.30	6.726	7.186	7.652	8.122	8.560	8.959	9.279	9.527	9.689	.000	.000
.35	7.074	7.504	7.932	8.356	8.744	9.092	9.368	9.579	9.717	.000	.000
.40	7.388	7.785	8.175	8.557	8.900	9.204	9.441	9.622	9.739	.000	.000
.45	7.674	8.038	8.391	8.732	9.034	9.299	9.503	9.658	9.758	.000	.000
.50	7.937	8.268	8.585	8.887	9.152	9.381	9.557	9.690	9.774	.000	.000
.55	8.183	8.480	8.761	9.027	9.256	9.454	9.604	9.717	9.789	.000	.000
.60	8.412	8.675	8.922	9.153	9.350	9.519	9.646	9.741	9.801	.000	.000
.65	8.627	8.857	9.070	9.267	9.435	9.577	9.683	9.763	9.813	.000	.000
.70	8.831	9.027	9.207	9.373	9.512	9.630	9.717	9.782	9.823	.000	.000
.75	9.024	9.187	9.335	9.471	9.583	9.678	9.748	9.800	9.833	.000	.000
.80	9.209	9.338	9.455	9.561	9.649	9.722	9.776	9.816	9.841	.000	.000
.85	9.384	9.481	9.568	9.646	9.710	9.763	9.802	9.831	9.849	.000	.000
.90	9.553	9.617	9.674	9.725	9.767	9.801	9.826	9.845	9.857	.000	.000
.95	9.714	9.746	9.774	9.799	9.820	9.837	9.849	9.858	9.863	.000	.000
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000	.000

TABLE II (CONTINUED)

		C/L=0.45										
		N=4.0										
		A/L										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11/12												
.05		3.761	4.315	4.976	5.754	6.640	7.585	8.477	9.173	9.598	9.795	.000
.10		4.764	5.349	6.012	6.746	7.520	8.275	8.926	9.403	9.686	9.818	.000
.15		5.449	6.024	6.656	7.326	8.001	8.626	9.144	9.512	9.729	9.830	.000
.20		5.983	6.535	7.125	7.733	8.324	8.855	9.282	9.581	9.756	9.837	.000
.25		6.427	6.950	7.496	8.044	8.565	9.021	9.381	9.630	9.775	9.843	.000
.30		6.811	7.300	7.802	8.296	8.755	9.150	9.458	9.668	9.790	9.847	.000
.35		7.150	7.605	8.063	8.507	8.912	9.255	9.519	9.698	9.802	9.850	.000
.40		7.456	7.874	8.291	8.687	9.044	9.342	9.569	9.723	9.812	9.853	.000
.45		7.734	8.117	8.492	8.845	9.158	9.417	9.613	9.744	9.820	9.855	.000
.50		7.991	8.337	8.673	8.984	9.258	9.482	9.650	9.763	9.827	9.857	.000
.55		8.229	8.539	8.837	9.109	9.347	9.539	9.683	9.779	9.834	9.859	.000
.60		8.452	8.727	8.987	9.223	9.426	9.590	9.712	9.793	9.839	9.861	.000
.65		8.662	8.901	9.125	9.326	9.499	9.637	9.738	9.806	9.844	9.862	.000
.70		8.860	9.064	9.253	9.422	9.565	9.678	9.762	9.817	9.849	9.864	.000
.75		9.048	9.217	9.372	9.509	9.625	9.717	9.784	9.828	9.853	9.865	.000
.80		9.227	9.361	9.484	9.591	9.681	9.752	9.804	9.838	9.857	9.866	.000
.85		9.398	9.498	9.588	9.667	9.733	9.785	9.822	9.846	9.861	9.867	.000
.90		9.562	9.628	9.687	9.739	9.782	9.815	9.839	9.855	9.864	9.868	.000
.95		9.719	9.751	9.781	9.806	9.827	9.843	9.855	9.862	9.867	9.869	.000
1.00		9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	.000

TABLE II (CONTINUED)

N=4.0 C/L=0.50

I1/I2	A/L										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.05	3.817	4.400	5.102	5.931	6.870	7.847	8.727	9.377	9.722	9.850	9.870
.10	4.812	5.420	6.113	6.879	7.680	8.446	9.083	9.534	9.767	9.856	9.870
.15	5.491	6.085	6.739	7.432	8.124	8.754	9.261	9.611	9.790	9.859	9.870
.20	6.020	6.589	7.197	7.821	8.424	8.957	9.374	9.660	9.805	9.861	9.870
.25	6.461	6.997	7.557	8.119	8.648	9.105	9.457	9.695	9.816	9.862	9.870
.30	6.841	7.341	7.855	8.360	8.825	9.220	9.520	9.722	9.824	9.864	9.870
.35	7.177	7.641	8.110	8.561	8.971	9.314	9.571	9.744	9.831	9.864	9.870
.40	7.479	7.906	8.331	8.734	9.095	9.392	9.614	9.762	9.836	9.865	9.870
.45	7.755	8.145	8.527	8.885	9.201	9.459	9.651	9.778	9.841	9.866	9.870
.50	8.009	8.362	8.703	9.019	9.295	9.518	9.682	9.791	9.845	9.866	9.870
.55	8.245	8.561	8.863	9.139	9.378	9.570	9.710	9.803	9.849	9.867	9.870
.60	8.466	8.745	9.009	9.248	9.453	9.616	9.735	9.813	9.852	9.867	9.870
.65	8.674	8.916	9.144	9.348	9.521	9.658	9.757	9.823	9.855	9.868	9.870
.70	8.870	9.077	9.268	9.439	9.583	9.696	9.778	9.831	9.858	9.868	9.870
.75	9.056	9.227	9.385	9.523	9.640	9.731	9.796	9.839	9.860	9.868	9.870
.80	9.233	9.369	9.493	9.602	9.692	9.763	9.813	9.846	9.862	9.869	9.870
.85	9.403	9.504	9.596	9.675	9.741	9.792	9.829	9.853	9.864	9.869	9.870
.90	9.565	9.632	9.692	9.744	9.787	9.820	9.843	9.859	9.866	9.869	9.870
.95	9.720	9.753	9.783	9.809	9.830	9.845	9.857	9.864	9.868	9.869	9.870
1.00	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870	9.870

4. Discussion of Results

In a preliminary investigation the factor k was varied from 20 to 160 in the program solution for a column which appears to have one of the most stringent requirements for correct convergence. It was assumed that an adequate value of k for this column is adequate for the remainder of the columns. The column is symmetric, i.e., $c/\ell = 0.5$; and $n = 4$, $I_1/I_2 = 0.10$, and $a/\ell = 0.0$. The following converged values of m were obtained for each k :

a) $k = 20$	$m = 4.81242$
b) $k = 40$	$m = 4.81206$
c) $k = 80$	$m = 4.81203$
d) $k = 160$	$m = 4.81203$

It is evident for the six significant figures shown, that when $k = 80$, the process converges to the proper value. The requirement is for only four significant figures, however, and $k = 40$ safely fulfills this requirement.

The final results in Table II were analyzed by: verifying that the solutions for prismatic columns, i.e., $a/\ell = 1.0$ or $I_1/I_2 = 1.0$, agree with theory; verifying that the results agree with the few analytical solutions for which there is a known deflection curve; and comparing the results with those buckling factors available from Dinnik's solutions.

For a uniform slender column with hinge supported ends, the buckling factor m is equal to π^2 . The buckling factors in Table II agree with the value of π^2 to four significant figures for all columns where either a/ℓ or I_1/I_2 is unity.

There are four analytical solutions which are both pertinent to this study and are readily obtainable:

- (1) $c/l = 0.0$, $n=2$, $a/l = 0.0$, $0 \leq I_1/I_2 < 1.0$
- (2) $c/l = 0.5$, $n=2$, $a/l = 0.0$, $0 \leq I_1/I_2 < 1.0$
- (3) $c/l = 0.0$, $n=4$, $a/l = 0.0$, $0 \leq I_1/I_2 < 1.0$
- (4) $c/l = 0.5$, $n=4$, $a/l = 0.0$, $0 \leq I_1/I_2 < 1.0$

These solutions were adapted from similar solutions by Bleich (12) and Timoshenko and Gere (13) and are presented in Appendix B. A comparison of the buckling factors obtained from them with those of identical columns from the numerical solution in this study is contained in Table III. The agreement between the two is excellent. The two non-symmetric cases have a maximum deviation between analytical and numerical results of 0.00002. This is quite good since the criterion for convergence in the numerical program was that the maximum and minimum values be within 0.0005 of each other. The maximum deviation of 0.00010 in the two symmetric cases is not as small, but is still well within an acceptable limit.

The buckling factors which are applicable from Dinnik's work were obtained from Timoshenko and Gere (13) and are listed in Table IV. Under each of these values is the corresponding value from this study. There is a small deviation in these entries in a large number of cases; $n = 1$ seems to be the prime offender. The maximum difference is 0.06. Since the present study agrees so closely with the exact solutions, and since the same method of solution is employed at all times, Dinnik's results seem to be in error by the amount of deviations from the present study.

TABLE III
ANALYTICAL CHECK OF THE CRITICAL BUCKLING FACTOR

N=2.0		C/L=0.0	A/L=0.0
I1/I2	NUMERICAL RESULTS	ANALYTICAL RESULTS	
.05	2.80237	2.80235	
.10	3.59827	3.59826	
.15	4.21168	4.21168	
.20	4.73361	4.73360	
.25	5.19807	5.19807	
.30	5.62218	5.62218	
.35	6.01594	6.01594	
.40	6.38581	6.38581	
.45	6.73624	6.73624	
.50	7.07045	7.07045	
.55	7.39085	7.39085	
.60	7.69931	7.69932	
.65	7.99733	7.99733	
.70	8.28609	8.28608	
.75	8.56657	8.56656	
.80	8.83959	8.83959	
.85	9.10586	9.10586	
.90	9.36595	9.36595	
.95	9.62039	9.62039	

N=4.0		C/L=0.0	A/L=0.0
I1/I2	NUMERICAL RESULTS	ANALYTICAL RESULTS	
.05	2.20692	2.20691	
.10	3.12105	3.12104	
.15	3.82248	3.82248	
.20	4.41382	4.41382	
.25	4.93480	4.93480	
.30	5.40581	5.40580	
.35	5.83894	5.83894	
.40	6.24209	6.24209	
.45	6.62073	6.62073	
.50	6.97886	6.97886	
.55	7.31949	7.31949	
.60	7.64496	7.64496	
.65	7.95713	7.95713	
.70	8.25751	8.25751	
.75	8.54733	8.54733	
.80	8.82764	8.82764	
.85	9.09933	9.09933	
.90	9.36313	9.36313	
.95	9.61971	9.61970	

TABLE III (CONTINUED)

N=2.0	C/L=0.5	A/L=0.0
I1/I2	NUMERICAL RESULTS	ANALYTICAL RESULTS
.05	4.64476	4.64466
.10	5.39883	5.39885
.15	5.93412	5.93414
.20	6.36478	6.36479
.25	6.73179	6.73187
.30	7.05536	7.05540
.35	7.34687	7.34688
.40	7.61359	7.61359
.45	7.86047	7.86045
.50	8.09101	8.09098
.55	8.30782	8.30779
.60	8.51291	8.51288
.65	8.70784	8.70781
.70	8.89386	8.89383
.75	9.07199	9.07196
.80	9.24308	9.24305
.85	9.40782	9.40781
.90	9.56683	9.56682
.95	9.72069	9.72060

N=4.0	C/L=0.5	A/L=0.0
I1/I2	NUMERICAL RESULTS	ANALYTICAL RESULTS
.05	3.81652	3.81648
.10	4.81206	4.81210
.15	5.49075	5.49078
.20	6.02032	6.02034
.25	6.46052	6.46061
.30	6.84050	6.84055
.35	7.17659	7.17662
.40	7.47914	7.47913
.45	7.75504	7.75503
.50	8.00926	8.00924
.55	8.24542	8.24540
.60	8.46626	8.46623
.65	8.67393	8.67390
.70	8.87014	8.87011
.75	9.05627	9.05624
.80	9.23345	9.23342
.85	9.40263	9.40261
.90	9.56461	9.56460
.95	9.72016	9.72007

TABLE IV
COMPARISON WITH DINNIK'S VALUES

I_1/I_2	n	a/l				
		0	0.2	0.4	0.6	0.8
0.1	1	6.48	7.58	8.63	9.46	9.82
		6.484	7.600	8.689	9.476	9.819
	2	5.40	6.67	8.08	9.25	9.79
		5.399	6.670	8.088	9.251	9.790
	3	5.01	6.32	7.84	9.14	9.77
		5.007	6.303	7.824	9.144	9.776
	4	4.81	6.11	7.68	9.08	9.77
		4.812	6.113	7.680	9.083	9.767
0.2	1	7.01	7.99	8.91	9.63	9.82
		7.009	7.999	8.916	9.555	9.829
	2	6.37	7.49	8.61	9.44	9.81
		6.365	7.487	8.610	9.444	9.814
	3	6.14	7.31	8.49	9.39	9.81
		6.136	7.296	8.488	9.399	9.808
	4	6.02	7.20	8.42	9.38	9.80
		6.020	7.197	8.424	9.374	9.805
0.4	1	7.87	8.59	9.19	9.70	9.84
		7.870	8.614	9.250	9.668	9.843
	2	7.61	8.42	9.15	9.63	9.84
		7.614	8.430	9.150	9.598	9.839
	3	7.52	8.38	9.12	9.62	9.84
		7.524	8.364	9.113	9.621	9.837
	4	7.48	8.33	9.10	9.62	9.84
		7.479	8.331	9.095	9.614	9.836

¹Upper values in each set of numbers are from Dinnik's work; those from the present work are directly below.

TABLE IV (CONTINUED)

I_1/I_2	n	a/ℓ				
		0	0.2	0.4	0.6	0.8
0.6	1	8.61	9.12	9.55	9.76	9.85
		8.604	9.100	9.500	9.751	9.854
	2	8.51	9.04	9.48	9.74	9.85
		8.513	9.040	9.469	9.740	9.853
	3	8.50	9.02	9.46	9.74	9.85
		8.482	9.020	9.458	9.737	9.852
	4	8.47	9.01	9.45	9.74	9.85
		8.466	9.009	9.453	9.735	9.852
0.8	1	9.27	9.54	9.69	9.83	9.86
		9.262	9.511	9.701	9.816	9.863
	2	9.24	9.50	9.69	9.82	9.86
		9.243	9.499	9.695	9.814	9.862
	3	9.23	9.50	9.69	9.81	9.86
		9.237	9.495	9.693	9.813	9.862
	4	9.23	9.49	9.69	9.81	9.86
		9.233	9.493	9.692	9.813	9.862

The results, as mentioned previously, cover the usable range for columns of the type under study. There might be a necessity, however, for obtaining intermediate values within this range which are not listed in Table II. Therefore, some consideration should be given to the method of interpolation. Since the data obtained form curves of second order or higher, an error is introduced when employing linear interpolation. The amount of error depends upon the position of the needed value in Table II. The error increases with increasing n , decreasing a/ℓ , and decreasing I_1/I_2 .

A comparison of linear interpolation with interpolation using Bessel's formula is given in Appendix D. Interpolation formulas and functions were obtained from Comrie (14). For a column with $c/\ell = 0.5$, $n = 4$, $I_1/I_2 = 0.23$, and $a/\ell = 0.15$, it was found that there is a 0.11% difference between linear and second degree interpolation.

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APPENDIX A

NEWMARK REFINEMENTS

Newmark (6) presents refinements which improve the accuracy of the Vianello method. He bases these refinements on the assumption that y'' is a second degree curve, whereas, in the basic method, it is assumed to be linear. He assumes that this curve takes the form of a distributed loading curve, Fig. 2(a), and then solves for the reactions at the end of each increment (λ). He finds that:

$$R_L = \frac{\lambda}{24} (7q + 6r - s) \quad (A.1)$$

$$R_{c_L} = \frac{\lambda}{24} (3q + 10r - s)$$

$$R_{c_R} = \frac{\lambda}{24} (3s + 10r - q)$$

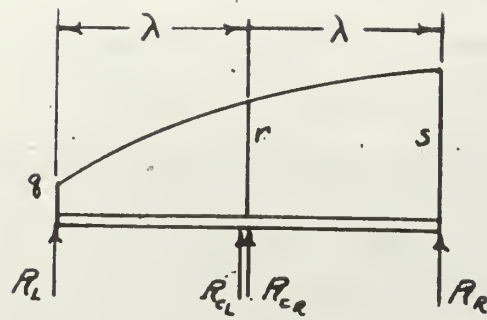
$$R_c = R_{c_L} + R_{c_R} = \frac{\lambda}{12} (q + 10r + s) \quad (A.2)$$

$$R_R = \frac{\lambda}{24} (7s + 6r - q)$$

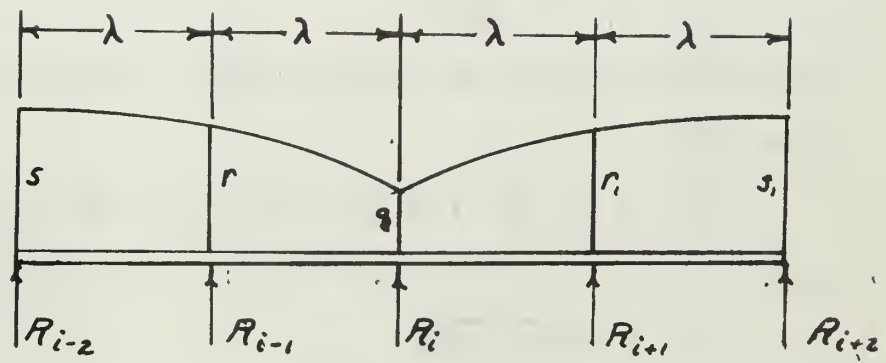
Equation (A.2) is used in the present study when the curve is smooth. When there is a break in the curve, equation (A.1) is used. The breaks in the curve appear where the tapered sections meet the prismatic section or each other. The curve is assumed to take the shape of Fig. 2(b) and the modification is:

$$R_i = \frac{\lambda}{24} (14q + 6r + 6r_i - s - s_i)$$

The reaction R is the concentrated angle change ($\Delta y'$).



(a)



(b)

Figure 2 Newmark Reactions

APPENDIX B

ANALYTICAL SOLUTIONS

(a) Symmetric Column, $n = 2$, $a/l = 0.0$, $0.0 \leq I_1/I_2 < 1.0$, $c/l = 0.5$

The column is of the form shown in Fig. 3, and:

$$I_x = I_2 \left(\frac{x^*}{j} \right)^2 = I_2 \xi^2$$

The differential equation of the deflection curve is:

$$EI_x \frac{d^2 y}{dx^{*2}} + Py = 0$$

or, substituting for I_x

$$\xi^2 \frac{d^2 y}{d\xi^{*2}} + \alpha^2 y = 0 \quad (B.1)$$

where

$$\alpha^2 = \frac{Pj^2}{EI_2}$$

The general solution of equation (B.1) with two arbitrary constants A and B is:

$$y = \sqrt{\xi} [A \sin(h \ln \xi) + B \cos(h \ln \xi)] \quad (B.2)$$

where

$$h = \sqrt{\alpha^2 - 1/4}$$

The boundary conditions for a symmetric hinged end column may be taken as:

$$\xi = \left(\frac{I_1}{I_2} \right)^{\frac{1}{2}}, \quad y = 0 \quad \text{at} \quad x = 0$$

$$\xi = 1, \quad \frac{dy}{d\xi} = 0 \quad \text{at} \quad x = l/2$$

Substituting these boundary conditions into equation (B.2)

results in two homogeneous equations for the constants A and B:

$$A \sin \left[h \ln \left(\frac{I_1}{I_2} \right)^{\frac{1}{2}} \right] + B \cos \left[h \ln \left(\frac{I_1}{I_2} \right)^{\frac{1}{2}} \right] = 0$$

$$Ah + B/2 = 0$$

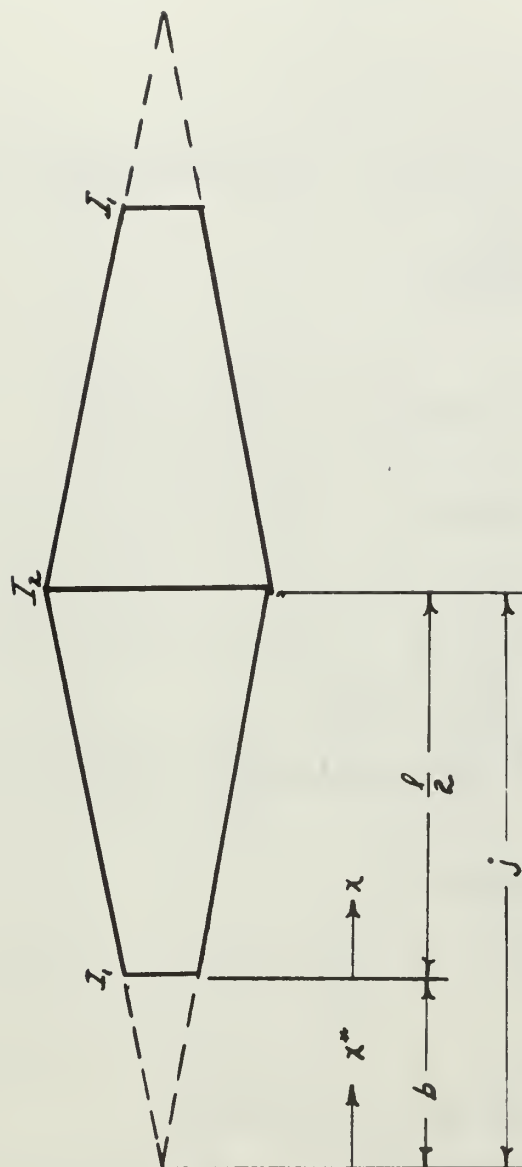


Figure 3 Symmetric Column
 $a/l = 0.0$

Solutions for A and B exist only if:

$$\tan \left[h \ln \left(\frac{I_1}{I_2} \right)^{\frac{1}{2}} \right] - 2h = 0$$

which has an infinite number of roots h . The smallest root h_1 defines the critical buckling load. This can be written as:

$$\frac{\tan \gamma}{\gamma} = \frac{4}{\ln(I_1/I_2)}$$

where

$$\gamma = \frac{1}{2} h_1 \ln(I_1/I_2)$$

With a given ratio of I_1/I_2 , γ can be obtained and the root h_1 is:

$$h_1 = \frac{2\gamma}{\ln(I_1/I_2)}$$

and

$$m = (1 + 4h_1^2) \left[1 - (I_1/I_2)^{\frac{1}{2}} \right]^2$$

(b) Non-Symmetric Column, $n = 2$, $a/l = 0.0$, $0.0 \leq I_1/I_2 < 1.0$,
 $c/l = 0.0$ or 1.0 .

The column is as shown in Fig. 4 and, as for the preceding column, the general solution is:

$$y = \sqrt{\xi} \left[A \sin(h \ln \xi) + B \cos(h \ln \xi) \right]$$

The boundary conditions are:

$$\xi = \left(\frac{I_1}{I_2} \right)^{\frac{1}{2}}, \quad y = 0 \quad \text{at } x = 0$$

$$\xi = 1.0, \quad y = 0 \quad \text{at } x = l$$

These conditions yield the two equations:

$$A \sin \left[h \ln \left(\frac{I_1}{I_2} \right)^{\frac{1}{2}} \right] + B \cos \left[h \ln \left(\frac{I_1}{I_2} \right)^{\frac{1}{2}} \right] = 0$$

$$B = 0$$

Therefore,

$$\sin \left[h \ln \left(\frac{I_1}{I_2} \right)^{\frac{1}{2}} \right] = 0$$

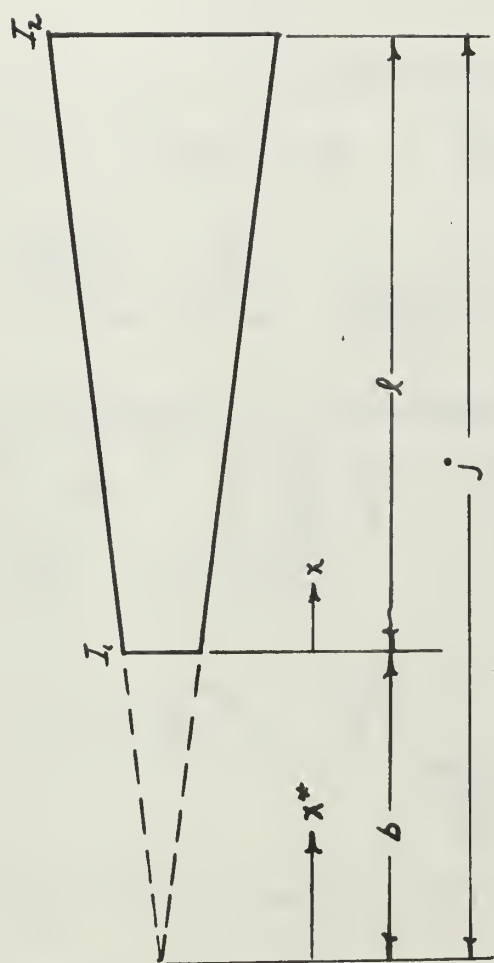


Figure 4 Non-symmetric Column
 $a/l=0.0$, $c/l=1.0$

or

$$h_1 = \frac{2\pi}{\ln(I_1/I_2)}$$

and

$$m = \left[1 - (I_1/I_2)^{\frac{1}{2}}\right]^2 \left\{ \frac{1}{4} + \left[\frac{2\pi}{\ln(I_1/I_2)} \right]^2 \right\}$$

(c) Symmetric Column, $n = 4$, $a/\ell = 0.0$, $c/\ell = 0.5$, $0.0 \leq I_1/I_2 \leq 1.0$.

The column is of the form known in Fig. 3 and:

$$I_x = I_2 \left(\frac{x^*}{j} \right)^4$$

The differential equation is:

$$EI_x \frac{d^2 y}{dx^{*2}} + Py = 0$$

or

$$\frac{EI_x}{j^4} x^{*4} \frac{d^2 y}{dx^{*2}} + Py = 0$$

substituting $t = 1/x^*$,

$$\frac{EI_2}{j^4} \left[\frac{d^2 y}{dt^2} + \frac{2}{t} \frac{dy}{dt} \right] + Py = 0$$

or

$$\frac{d^2 y}{dt^2} + \frac{2}{t} \frac{dy}{dt} + \delta^2 y = 0 \quad (B.3)$$

where

$$\delta^2 = \frac{Pj^4}{EI_2}$$

Equation (B.3) is a form of Bessel's differential equation and has the solution:

$$y = t^{-\frac{1}{2}} \left[A, J_{-\frac{1}{2}}(\delta t) + B, Y_{-\frac{1}{2}}(\delta t) \right]$$

where:

$$J_{-\frac{1}{2}}(\delta t) = \frac{\cos(\delta t)}{\sqrt{\pi \delta t/2}}$$

$$Y_{-\frac{1}{2}}(\delta t) = \frac{\sin(\delta t)}{\sqrt{\pi \delta t/2}}$$

so:

$$y = \frac{1}{\ell} \sqrt{\frac{2}{\pi \delta}} \left[A_1 \cos(\delta t) + B_1 \sin(\delta t) \right]$$

and the general solution is:

$$y = \chi^* \left[A_2 \cos\left(\frac{\delta}{\chi^*}\right) + B_2 \sin\left(\frac{\delta}{\chi^*}\right) \right] \quad (B.4)$$

The boundary conditions are:

$$\begin{aligned} \chi^* &= b, & y &= 0 & \text{at } \chi &= 0 \\ \chi^* &= j, & \frac{dy}{d\chi^*} &= 0 & \text{at } \chi &= \ell/2 \end{aligned}$$

Substituting these boundary conditions into Equation (B.4) and solving for A_2 and B_2 yields:

$$\tan\left(\frac{\delta \ell}{2bj}\right) = -\frac{\delta}{\ell}$$

or

$$\frac{\tan \beta}{\beta} = -\frac{b}{\ell}$$

where

$$\beta = \frac{\delta \ell}{2bj} = \frac{\ell}{2b} \sqrt{\frac{Pj^2}{EI_2}}$$

Then,

$$m = 4\beta^2(I_1/I_2)^{\frac{1}{2}}$$

(d) Non-Symmetric Column, $n = 4$, $a/\ell = 0.0$, $c/\ell = 0.0$ or 1.0 ,
 $0.0 \leq I_1/I_2 < 1.0$.

The column is of the form shown in Fig. 4 and the general solution is the same as for the preceding column:

$$y = \chi^* \left[A_2 \cos\left(\frac{\delta}{\chi^*}\right) + B_2 \sin\left(\frac{\delta}{\chi^*}\right) \right] \quad (B.4)$$

The boundary conditions are:

$$x^* = b, \quad y = 0 \quad \text{at} \quad x = 0$$

$$x^* = j, \quad y = 0 \quad \text{at} \quad x = l$$

Substituting these boundary conditions into Equation (B.4) and solving for A_2 and B_2 yields:

$$\sin\left(\frac{\delta l}{bj}\right) = 0$$

which has an infinite number of roots $n\pi$. The smallest non-trivial root π defines the critical buckling load. Then,

$$\frac{\delta l}{bj} = \pi$$

and

$$m = \pi^2 \left(\frac{I_1}{I_2} \right)^{\frac{1}{2}}$$

APPENDIX C

PROGRAM LISTING

```

..JOB0499F,BRANCH
PROGRAM NEWMARK
DIMENSION X(110), Y(110), YYY(110), RAT(110), T(110), RATIO(2
11), CRLOAD(21,12), ADJ(110)
PRINT 300
300 FORMAT (1H1/////////44X$HTABLE 11/36X24HCRITICAL BUCKLING FACTOR/
1)
DO 112 IC=0,10
TERM=IC
CAOL=TERM/20.0
ICE=IC+1
DO 111 IE=1,4
EXP=IE
PRINT 304, (EXP, CAOL)
304 FORMAT (1H 35X2HN=F3.1,12X5HC/L=0F3.2/)
PRINT 302
302 FORMAT (1H 45X3HA/L)
PRINT 303
303 FORMAT (1H 9X3H0.05X3H0.15X3H0.25X3H0.35X3H0.45X3H0.55X3H0.65X3H0.
175X3H0.85X3H0.95X3H1.0)
PRINT 305
305 FORMAT (1H 2X5H11/12)
KAK=5
DO 111 11=1,20
F=11
RATIO(11)=F/20.0
P=(RATIO(11))* (1.0/EXP)
DO 110 N1=1,ICE
A=N1-1
AOL=A/10.0
COL=CAOL-AOL/2.0
DOL=1.0-(COL+AOL)
IF (1.0-RATIO(11)) 17,17,18
18 DOL=(P/(1.0-P))*COL
EOL=(P/(1.0-P))*DOL
17 N=4

```



```

K1=K+1
36 DO 40 J=1,K1
   ITER=1
   B=J I
   C=K
   XOL=B/C
   IF(1.0-RATIO(II)) 5,5,19
19 IF (1.0-AOL) 5,5,16
16 IF (XOL-COL) 2,3,3
2  RAT(J)=((XOL+BOL)/(BOL+COL))*EXP
   GO TO 7
3  IF(XOL-(COL+AOL)) 5,6,6
5  RAT(J)=1.0
   GO TO 7
6  RAT(J)=((1.0+EOL-XOL)/(DOL+EOL))*EXP
7  U=1.0-2.0*(XOL)
40 X(J)=(1.0-U*U)*(1.0-RATIO(II)) + (SINF(3.1415927*XOL))*RATIO(II)
   K2=COL*C+1.0
   K3=AOL*C
4  DO 50 I=1,K1
50 ADJ(I)=-X(I)/(RAT(I))
   YYY(1)=0.0
   DO 62 N=2,K
   IF (N-K2) 63,64,65
65 IF (N-(K2+K3)) 63,64,63
64 YYY(N)=(14.*ADJ(N)+6.*(ADJ(N+1)+ADJ(N-1))-ADJ(N+2)-ADJ(N-2))/24.0
   GO TO 62
63 YYY(N)=(ADJ(N-1)+10.0*(ADJ(N)+ADJ(N+1))/12.0
62 CONTINUE
   YY(K/2)=-YYY(K/2+1)/2.0
   DO 20 L=1,K
   IF(L-K/2) 8,20,10
8  YY(K/2-L)=YY(K/2-L+1)-YYY(K/2-L+1)
   GO TO 20
10 YY(L)=YY(L-1)+YYY(L)
20 CONTINUE

```

```

Y(1)=0.0
DO 21 M=2,K1
21 Y(M)=Y(M-1)+YY(M-1)
IF (Y(K+1)) 22,25,22
22 DINC=-(Y(K+1))/C
DO 24 N=1,K1
D=N 1
24 Y(N)=Y(N)+DINC*D
23 DO 25 KL=2,K
T(KL)=(X(KL)*C*C)/Y(KL)
IF (2-KL) 26,27,27
27 Z=T(2)
S=T(2)
26 IF (2-T(KL)) 28,29,29
28 Z=T(KL)
29 IF (T(KL)-S) 30,25,25
30 S=T(KL)
25 CONTINUE
IF(Z-S-0.0005) 32,32,31
31 DO 33 JK=1,K1
33 X(JK)=Y(JK)
60 GO TO 4
32 CRLOAD(11,NI)=(Z+S)/2.0
110 CONTINUE
IF (11-KAK) 111,111,113
113 KAK=KAK+5
PRINT 306
306 FORMAT (1H )
111 PRINT 61,(RATIO(11),CRLOAD(11,1),CRLOAD(11,2),CRLOAD(11,3),CRLOAD(
111,4),CRLOAD(11,5),CRLOAD(11,6),CRLOAD(11,7),CRLOAD(11,8),CRLOAD(1
21,9),CRLOAD(11,10),CRLOAD(11,11))
61 FORMAT (1H 1XF4.2,11F8.3)
112 PRINT 308
308 FORMAT (1H1//////////38X20HTABLE11 (CONTINUED)/)
PRINT 307
307 FORMAT (1H1)
END

```

APPENDIX D

INTERPOLATION METHODS

A value of m is desired for $c/\ell = 0.5$, $n = 4$, $I_1/I_2 = 0.23$, and $a/\ell = 0.15$.

(a) Second degree interpolation

Bessel's formula is modified to:

$$m = m_0 + p\Delta_{\frac{1}{2}}' + \frac{p(p-1)}{4} (\Delta_0'' + \Delta_1'')$$

where it is assumed that the third differences are negligible.

$$I_1/I_2 = 0.15, a/\ell = 0.15$$

a/ℓ	m	Δ'	Δ''
0.0	5.491	594	
0.1	6.085	654	60
0.2	6.739	693	39
0.3	7.432		

$$m = 6.085 + 0.5 (.654) - .062 (.099) = 6.406$$

$$I_1/I_2 = 0.20, a/\ell = 0.15$$

a/ℓ	m	Δ'	Δ''
0.0	6.020	569	
0.1	6.589	608	39
0.2	7.197	624	16
0.3	7.821		

$$m = 6.589 + 0.5 (.608) - .062 (.055) = 6.890$$

$$I_1/I_2 = 0.25, a/\ell = 0.15$$

a/ℓ	m	Δ'	Δ''
0.0	6.461	536	
0.1	6.997	560	24
0.2	7.557	562	2
0.3	8.119		

$$m = 6.997 + 0.5 (.560) - .062 (.026) = 7.275$$

$$I_1/I_2 = 0.30, a/\ell = 0.15$$

a/ℓ	m	Δ'	Δ''
0.0	6.841	500	
0.1	7.341	514	14
0.2	7.855	505	-9
0.3	8.360		

$$m = 7.341 + 0.5 (.514) - .062 (.005) = 7.598$$

$$I_1/I_2 = 0.23, a/\ell = 0.15$$

I_1/I_2	m	Δ'	Δ''
0.15	6.406	484	
0.20	6.890	385	-99
0.25	7.275	323	-62
0.30	7.598		

$$m = 6.890 + 0.6 (.385) - .060 (.161) = 7.131$$

(b) Linear Interpolation

$$I_1/I_2 = 0.20, a/\ell = 0.15$$

$$m = 6.589 + .304 = 6.893$$

$$I_1/I_2 = 0.25, a/\ell = 0.15$$

$$m = 6.997 + 0.6 (.560) = 7.277$$

$$I_1/I_2 = 0.23, a/\ell = 0.15$$

$$m = 6.893 + 0.6 (.384) = 7.123$$

(c) Comparison between the two methods

$$\text{difference} = \frac{7.131 - 7.123}{7.131} = 0.11\%$$

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KEY WORDS

LINK A

LINK B

LINK C

ROLE

WT

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